GEAR SHAPED CUTTER – A PROFILING METHOD DEVELOPED IN GRAPHICAL FORM

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ABSTRACT

The classical methods for the study of enveloping surfaces are known as basic methods, such as Olivier theorems, Gohman method or Willis theorem. Other methods are known as specific methods, as, for example, the Nikolaev method for profiling tools which generate helical surfaces, or complementary methods for the study of enveloping surfaces.

This paper presents an alternative method – the method of relative trajectories – for profiling tools which generate by enveloping ordered curls of profiles associated with a pair of rolling centrodes. An analytical argument is presented, for the study of generation using gear shaped cutter as well as a graphical method developed in CATIA design environment, using capabilities of this software. Numerical applications are also presented, including a comparison between the results obtained with an analytical method and the results obtained with the graphical method, in order to prove the quality of the proposed method.

KEYWORDS: gear shaped cutter; relative trajectories, curves enveloping, CATIA

1. INTRODUCTION

The generation of ordered curls profiles, such as teethed wheels flanks, teeth of side mill for cutting, polygonal shafts, or polygonal bushes, is usually machined with gear shaped cutters. In this case, the profiles of the curls to be generated and, in the same time, the profiles of the tool's teeth are associated with a pair of rolling centrodes. In most cases these centrodes are circles.

The issue is related to determination the profile for the gear shaped cutter teeth. The tools generate by shaping, in the rolling process, an ordered curl of profiles associate with a centrode.

The problem can be solved using the Willis basic theorem of gearing [1], also, using the Gohman theorem [2], [3], using complementary theorems, as the "minimum distance" method or the method of "in-plane generating trajectories" [2], [3]. Moreover, the graphical methods were realised in AutoCAD [4]. In these methods the 2D modelling is used and rigorous solutions are presented, based on an original software programmed in Lisp.

Using the CATIA graphical environment, "generating mechanisms" were imagined for the profile to be generated in order to find profiling solutions for the gear shaped cutter tools [5].

The problems of profiles interference are examined.

In this paper is proposed a graphical solution, based on the description of the generating relative trajectories (in principle, cycloid curves) of points belongs to the profile to be generated in the tool's space. A new form for enveloping condition is proposed. The solution is simple, easy to apply and rigorous.

Comparative applications are presented versus analytical solution in order to prove the method's quality.

An assembly composed of three elements is used in order to make the graphical profiling.

The first element represents the tool's centrode, the circle with radius R_{r2}, the axis, and the $X_1Y_1Z_1$ reference system's origin.

The second element is the piece's centrode and the reference system joined with this. Moreover, the profile to be generated is sketched here, and, also the straight lined segment perpendicularly to this profile.

The third element is named "*SR_fix*" and includes the fixed reference systems. This element is designed to ensure the relative positioning between piece and tool.

The gearing pole is defined in this third reference system.

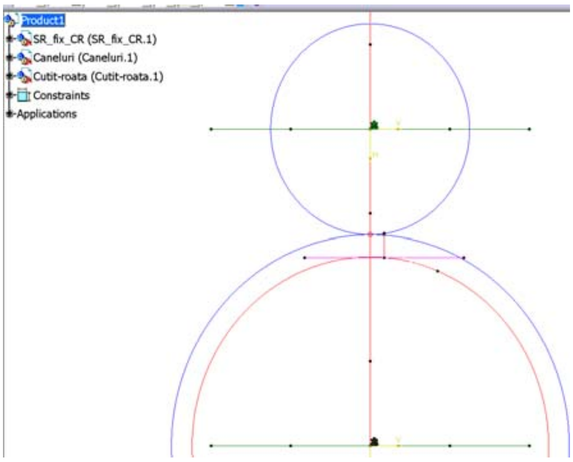

Fig. 1. *The three elements of the assembly*

A gear type mechanism is defined in the assembly shown above. This mechanism has, as fixed part, the tool and, as driving element, the rotation angle of the reference system joined with the tool. Here is measured and keep the distance from gearing pole to the normal line drawn to the profile.

The following algorithm is applied in order to identify the contact points between piece and tool:

1). The position where the normal to the profile pass through the gearing pole is identified simulating the mechanism. In practice this identification is made monitoring the value of the above mentioned distance and finding the mechanism position when this distance is below a certain value (10^{-3}÷10^{-4} mm).

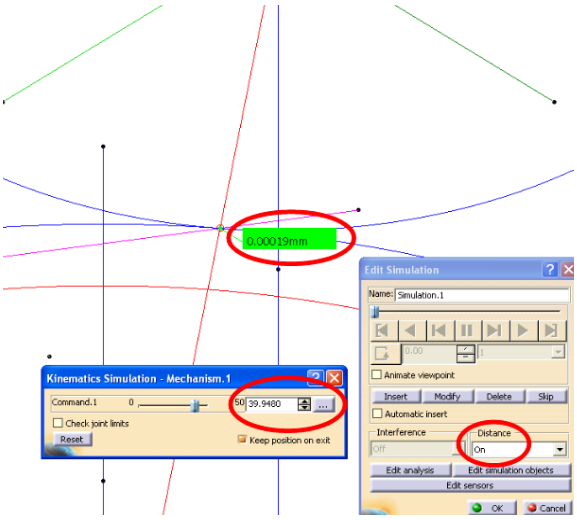

Fig. 2. *Distance between gear pole and normal; followed elements*

2). With the mechanism in this position, the sketch of the piece’s profile is opened and the intersection point between the normal and the piece’s profile is fixed. For a correct positioning there are temporary applied constraints of this point, so it belongs simultaneously to the normal and to the profile. Subsequently, these constraints are deleted in order not to modify the position of this point.

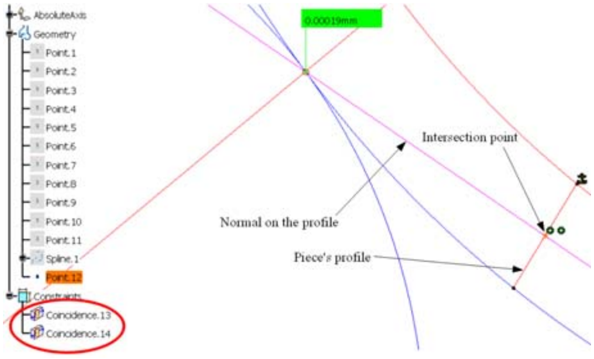

Fig. 3. *Intersection point between normal and profile; temporary constraints*

3). The piece draw is opened and the normal to the profile is moved into a new position of the current point.

Step 1 is resumed with the new position of the normal and is continued with steps 2 and 3.

After an adequate number of points are identified on the tool’s profile, this may be materialized by drawing a spline curve through these points.

2. GENERATION OF RELATIVE TRAJECTORIES IN ROLLING PROCESS WITH GEAR SHAPED CUTTER

The pair of centrodes is presented in figure 4. These centrodes are circles, with Rr1 and Rr2 radii, for the rotary tool and blank. The reference systems are defined as:

- *xy* is the global reference system, joined with the revolution axis of the blank and the C_1 centrode;

- x_0y_0 — global reference system, joined with the C_2 centrode’s axis;

- *XY* — mobile reference system, joined with the profile to be generated, initially overlapped on the *xy* reference system;

- X_1Y_1 — mobile reference system, joined with the future tool. The rolling process kinematics for generation with gear shaped cutter assumes to accept the equivalence:

$$R_{r1}\cdot\varphi_1 = R_{r2}\cdot\varphi_2 . \qquad (1)$$

Equation (1) represents the rolling motion between the two centrodes, C_1 and C_2. The global motions are also known:

$$x = \omega_3^T(\varphi_1)\cdot X \qquad (2)$$

and

$$x_0 = \omega_3^T(-\varphi_2)\cdot X_1, \quad (3)$$

of the two centrodes and, at the same time, of the spaces associated with XY and X_1Y_1.

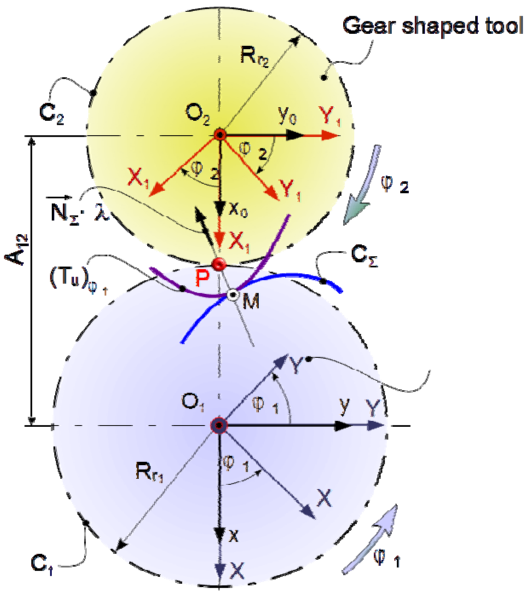

Fig. 4. *Rolling centrodes; reference systems; C_Σ profile*

The position of the global reference systems is known:

$$x_0 = x - a;\ a = \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix};\ A_{12} = R_{r1} + R_{r2}. \quad (4)$$

Now, the relative motion of the XY space regarding the X_1Y_1 space can be deduced,

$$X_1 = \omega_3(-\varphi_2)\cdot\left[\omega_3^T(\varphi_1)\cdot X\text{-}a\right]. \quad (5)$$

The ordered curl of profiles is defined as:

$$C_\Sigma \begin{vmatrix} X = X(u); \\ Y = Y(u), \end{vmatrix} \text{ or } \vec{r} = X(u)\cdot\vec{i} + Y(u)\cdot\vec{j}, \quad (6)$$

so the directrix parameters of the normal to the profile are defined,

$$\vec{N}_{C_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dot{X}_u & \dot{Y}_u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \dot{Y}_u\cdot\vec{i} - \dot{X}_u\cdot\vec{j} \quad (7)$$

or

$$\vec{N}_{C_\Sigma} = N_X\cdot\vec{i} + N_Y\cdot\vec{j}; \quad N_X = \dot{Y}_u;\ N_Y = -\dot{X}_u. \quad (8)$$

In this way, the vector of normal to C_Σ, in the XY space, has the form:

$$X = X(u) + \lambda\cdot N_X; \quad Y = Y(u) + \lambda\cdot N_Y. \quad (9).$$

The trajectories of normal to the C_Σ profile can be defined, regarding the gear shaped cutter reference system, X_1Y_1:

$$(N_{C_\Sigma})_{\varphi_1} \begin{vmatrix} X_1 = \left[X(u) + \lambda\cdot\dot{Y}_u\right]\cdot cos(\varphi_1+\varphi_2) - \\ -\left[Y(u) - \lambda\cdot\dot{X}_u\right]\cdot sin(\varphi_1+\varphi_2) + \\ A_{12}\cdot cos\,\varphi_2; \\ Y_1 = \left[X(u) + \lambda\cdot\dot{Y}_u\right]\cdot sin(\varphi_1+\varphi_2) + \\ +\left[Y(u) - \lambda\cdot\dot{X}_u\right]\cdot cos(\varphi_1+\varphi_2) + \\ A_{12}\cdot sin\,\varphi_2; \end{vmatrix} \quad (10)$$

and

$$\varphi_2 = R_{r_1}/R_{r_2}\cdot\varphi_1 \text{ or } \varphi_2 = i\cdot\varphi_1, \quad (11)$$

with i gear ratio.

For $\lambda = 0$, equations (10) represent the trajectories of points on the C_Σ in the relative motion regarding the gear shaped cutter reference system X_1Y_1:

$$(T_u)_{\varphi_1} \begin{vmatrix} X_1 = X(u)\cdot cos(1+i)\cdot\varphi_1 - \\ -Y(u)\cdot sin(1+i)\cdot\varphi_1 + A_{12}\cdot cos(i\cdot\varphi_1); \\ Y_1 = X(u)\cdot sin(1+i)\cdot\varphi_1 + \\ +Y(u)\cdot cos(1+i)\cdot\varphi_1 + A_{12}\cdot sin(i\cdot\varphi_1). \end{vmatrix} \quad (12)$$

The enveloping of the in-plane trajectories family $(T_u)_{\varphi_1}$, given by equations (12), represents the profile of the gear shaped cutter, reciprocally enveloping with the C_Σ ordered curl of profiles.

2.1. Enveloping condition

The specific enveloping condition is obtained from constraint that the normals' family (10) pass through the gearing pole (Willis theorem) [1].

The coordinates of the gearing pole (figure 1), in the $X_1Y_1Z_1$ reference system, are:

$$P \begin{vmatrix} X_1 = R_{r2}\cdot cos\,\varphi_2 = R_{r2}\cdot cos(i\cdot\varphi_1); \\ Y_1 = R_{r2}\cdot sin\,\varphi_2 = R_{r2}\cdot sin(i\cdot\varphi_1). \end{vmatrix} \quad (13)$$

In this way, the coincidence condition between the $(N_{C_\Sigma})_{\varphi_1}$ normals' family and gearing pole P, if the directrix parameters of the $\vec{N}_{C_\Sigma}$ normal, given by (7), are:

$$\begin{cases} \left[X(u)+\lambda\cdot\dot{Y}_u\right]\cdot cos\left(1+i\right)\cdot\varphi_1 - \\ -\left[Y(u)+\lambda\cdot\dot{X}_u\right]\cdot sin\left(1+i\right)\cdot\varphi_1 + \\ +A_{12}\cdot cos\left(i\cdot\varphi_1\right)=R_{r_2}\cdot cos\left(i\cdot\varphi_1\right); \\ \left[X(u)+\lambda\cdot\dot{Y}_u\right]\cdot sin\left(1+i\right)\cdot\varphi_1 + \\ +\left[Y(u)-\lambda\cdot\dot{X}_u\right]\cdot cos\left(1+i\right)\cdot\varphi_1 + \\ +A_{12}\cdot sin\left(i\cdot\varphi_1\right)=R_{r_2}\cdot sin\left(i\cdot\varphi_1\right), \end{cases} \quad (14)$$

which, removing the λ scalar parameter, determines the specific enveloping condition:

$$\frac{-X(u)cos\left(1+i\right)\varphi_1+Y(u)sin\left(1+i\right)\varphi_1-R_{r1}cos\left(i\varphi_1\right)}{\dot{Y}_u cos\left(1+i\right)\varphi_1+\dot{X}_u sin\left(1+i\right)\varphi_1}= \quad (15)$$

$$=\frac{X(u)sin\left(1+i\right)\varphi_1+Y(u)cos\left(1+i\right)\varphi_1+R_{r1}sin\left(i\varphi_1\right)}{\dot{X}_u cos\left(1+i\right)\varphi_1-\dot{Y}_u sin\left(1+i\right)\varphi_1}$$

where we consider the definition of the A_{12} value, the distance between the centrodes' axis, $A_{12}=R_{r1}+R_{r2}$. Finally, equation (15) can be brought to form:

$$\dot{X}_u cos\,\varphi_1-\dot{Y}\,sin\,\varphi_1=\frac{X\left(u\right)\dot{X}_u+Y\left(u\right)\dot{Y}_u}{-R_{r1}} \quad (16)$$

In this way, the (12) and (15) equation assemblies allow to determine the enveloping of the in-plane generating trajectories by removing one of the independent parameters u or φ_1. Essentially, this allows determining the profile of the gear shaped cutter tooth's flank.

In principle, condition (15) can be expressed in the form

$$u=u\left(\varphi_1\right), \quad (17)$$

and equations (12) have the form:

$$C_S\begin{vmatrix} X_1=X_1(u), \\ Y_1=Y_1(u). \end{vmatrix} \quad (18)$$

2.2. The gearing line

Obviously, the equations of the gearing line can be determined between the curves C_Σ (6) and C_S (18), associating the enveloping condition with a trajectory in one of the global reference systems, as the example in (2), resulting the form

$$\begin{Vmatrix} x \\ y \end{Vmatrix}=\begin{Vmatrix} cos\,\varphi_1 & -sin\,\varphi_1 \\ sin\,\varphi_1 & cos\,\varphi_1 \end{Vmatrix}\cdot\begin{Vmatrix} X(u) \\ Y(u) \end{Vmatrix} \quad (19)$$

to which is associated condition (17).

3. IN-PLANE GENERATING TRAJECTORIES IN THE GENERATING PROCESS WITH INTERNAL GEAR SHAPED CUTTERS

In figure 5 are presented the rolling centrodes in the generating process of the internal teeth-rolling centrode for internal gearing.

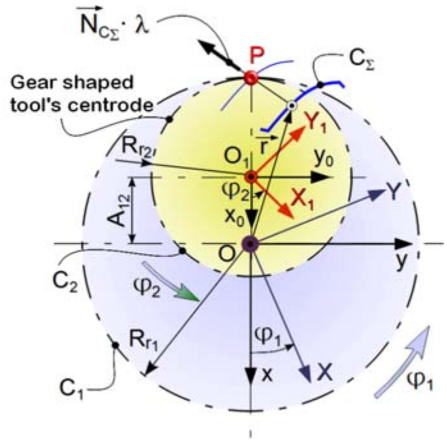

Fig. 5. *Internal gearing; centrodes C_1 and C_2; C_Σ profile to be generated*

The reference systems are defined:

- xy and x_0y_0 are global reference systems joined with axis of C_1 and respectively C_2 centrodes;
- XY — mobile reference system, joined with the generated profile, C_Σ;
- X_1Y_1 — mobile reference system, joined with the internal gear shaped cutter.

The distance between axes O and O_1 is defined as:

$$A_{12}=R_{r1}\text{-}R_{r2}\,. \quad (20)$$

The profile of the ordered curl of profiles is known in the XY mobile reference system,

$$C_\Sigma\begin{vmatrix} X=X(u); \\ Y=Y(u), \end{vmatrix} \quad (21)$$

with the variable parameter u.

The generating process kinematics includes the absolute motions:

$$x=\omega_3^T\left(\varphi_1\right)\cdot X \quad (22)$$

the revolving motion of the C_1 centrode;

$$x_0=\omega_3^T\left(\varphi_2\right)\cdot X_1 \quad (23)$$

the revolving motion of the C_2 centrode;

The rolling correlation can be established between the angular parameters φ_1 and φ_2, in form:

$$R_{r1}\cdot\varphi_1=R_{r2}\cdot\varphi_2 \text{ or } i=\frac{\varphi_2}{\varphi_1}=\frac{R_{r1}}{R_{r2}}, \quad (24)$$

where i is the gearing ration.

The relative position between the reference systems can be established:

$$x_0 = x - a; \quad a = \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix}, \qquad (25)$$

with A_{12} give by relation (17).

The relative motion between the mobile reference systems, similarly with motion (5), is described by the transformation

$$X_1 = \omega_3(\varphi_2) \cdot \left[\omega_3^T(\varphi_1) \cdot X\text{-}a \right], \qquad (26)$$

resulting

$$(T_{(u)})_{\varphi_1} \left| \begin{aligned} & X_1 = X(u) \cdot cos(i-1) \cdot \varphi_1 + \\ & + Y(u) \cdot sin(i-1) \cdot \varphi_1 + \\ & + A_{12} \cdot cos(i \cdot \varphi_1); \\ & Y_1 = -X(u) \cdot sin(i-1) \cdot \varphi_1 + \\ & + Y(u) \cdot cos(i-1) \cdot \varphi_1 + \\ & + A_{12} \cdot sin(i \cdot \varphi_1). \end{aligned} \right. \qquad (27)$$

The normals’ family to C_Σ can be written, starting from the equation (10):

$$(N_{C_\Sigma})_{\varphi_1} \left| \begin{aligned} & X_1 = \left[X(u) + \lambda \cdot \dot{Y}_u \right] \cdot cos(i-1) \cdot \varphi_1 + \\ & + \left[Y(u) - \lambda \cdot \dot{X}_u \right] \cdot sin(i-1) \cdot \varphi_1 + \\ & + A_{12} \cdot cos(i \cdot \varphi_1); \\ & Y_1 = -\left[X(u) + \lambda \cdot \dot{Y}_u \right] \cdot sin(i-1) \cdot \varphi_1 + \\ & + \left[Y(u) - \lambda \cdot \dot{X}_u \right] \cdot cos(i-1) \cdot \varphi_1 - \\ & - A_{12} \cdot sin(i \cdot \varphi_1). \end{aligned} \right. \qquad (28)$$

The coordinates of the gearing pole are:

$$\left| \begin{aligned} & X_{1P} = -R_{r2} \cdot cos(i \cdot \varphi_1); \\ & Y_{1P} = R_{r2} \cdot sin(i \cdot \varphi_1). \end{aligned} \right. \qquad (29)$$

Equations (29) and (28) allow determining the enveloping condition:

$$(N_{C_\Sigma})_{\varphi_1} \left| \begin{aligned} & \left[X(u) + \lambda \cdot \dot{Y}_u \right] cos(i-1) \cdot \varphi_1 + \\ & + \left[Y(u) - \lambda \cdot \dot{X}_u \right] sin(i-1)\varphi_1 + \\ & + A_{12} cos(i\varphi_1) = -R_{r_2} cos(i\varphi_1); \\ & - \left[X(u) + \lambda \cdot \dot{Y}_u \right] sin(i-1)\varphi_1 + \\ & + \left[Y(u) - \lambda \cdot \dot{X}_u \right] cos(i-1)\varphi_1 - \\ & - A_{12} sin(i\varphi_1) = R_{r_2} sin(i\varphi_1), \end{aligned} \right. \qquad (30)$$

by removing the λ parameter from the equation assembly (30),

$$\lambda = \frac{X(u)cos(i-1)\varphi_1 + Y(u)sin(i-1)\varphi_1 + R_{r1} cos(i\varphi_1)}{\dot{X}_u sin(i-1)\varphi_1 - \dot{Y}_u cos(i-1)\varphi_1} = \qquad (31)$$
$$= \frac{-X(u)sin(i-1)\varphi_1 + Y(u)cos(i-1)\varphi_1 - R_1 sin(i\varphi_1)}{\dot{X}_u cos(i-1)\varphi_1 + \dot{Y}_u sin(i-1)\varphi_1}$$

The enveloping condition and the generating trajectories family (27) represent, by removing one of the u or φ_1 parameters, the C_S profile of the gear shaped cutter, in principle, in the form:

$$C_S \left| \begin{aligned} & X_1 = X_1(u), \\ & Y_1 = Y_1(u). \end{aligned} \right. \qquad (32)$$

4. GEAR SHAPED CUTTER FOR GENERATING A SPLINED SHAFT

An application related to profiling the gear shaped cutter for machining a splined shaft is presented.

The shaft profile is shown in figure 6 and the flank equations are:

$$C_\Sigma \left| \begin{aligned} & X = -u; \\ & Y = a, \end{aligned} \right. \qquad (33)$$

with variable u.

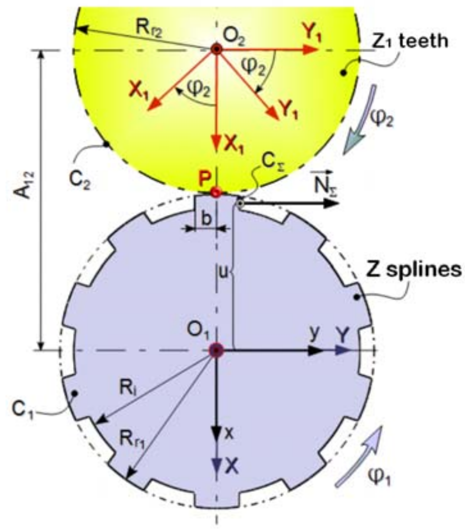

Fig. 6. *Splined shaft; reference systems*

The variation limits for parameter u are:

$$u_{min} = \sqrt{R_i^2 - a^2}\,; \; u_{max} = \sqrt{R_e^2 - a^2}\,. \qquad (34)$$

The unitary vector to the flank normal has equation

$$\vec{N}_\Sigma = \vec{j}\ . \qquad (35)$$

The equation of the normal to the C_Σ is

$$\vec{N}_{C_\Sigma} = -u \cdot \vec{i} + (a + \lambda) \cdot \vec{j}\,. \qquad (36)$$

In this way, the normals’ family $(N_{C_\Sigma})_{\varphi_1}$ has equations:

$$(N_{C_\Sigma})_{\varphi_1}\begin{vmatrix} X_1 = -u\cos(1+i)\varphi_1 - \\ -(a+\lambda)\sin(1+i)\varphi_1 + A_{12}\cos(i\varphi_1); \\ Y_1 = -u\sin(1+i)\varphi_1 + \\ +(a+\lambda)\cos(1+i)\varphi_1 + A_{12}\sin(i\varphi_1); \end{vmatrix} \quad (37)$$

with

$$i = \frac{\varphi_2}{\varphi_1}. \quad (38)$$

From the condition that the normals' family (37) pass through the gearing pole, *P*,

$$P\begin{vmatrix} X_1 = R_{r2}\cdot cos(i\cdot\varphi_1); \\ Y_1 = R_{r2}\cdot sin(i\cdot\varphi_1), \end{vmatrix} \quad (39)$$

results the enveloping condition,

$$\varphi_1 = arccos\left(\frac{u}{R_{r1}}\right). \quad (40)$$

The in-plane generating trajectories family results from (37) for $\lambda = 0$:

$$\begin{aligned} &X_1 = -u\cdot cos(1+i)\cdot\varphi_1 - a\cdot sin(1+i)\cdot\varphi_1 + \\ &+A_{12}\cdot cos(i\cdot\varphi_1); \\ &Y_1 = -u\cdot sin(1+i)\cdot\varphi_1 + a\cdot cos(1+i)\cdot\varphi_1 + \\ &+A_{12}\cdot sin(i\cdot\varphi_1). \end{aligned} \quad (41)$$

Equations (41) and (40) determine the gear shaped tool's profile.

4.1. Numerical application

A numerical application is presented for a splined shaft with *Re* = 62.5 mm; *Ri* = 56 mm; *z* = 20 splines; *b*=4.5 mm; radius of gear shaped tool R_{r2} = 31.25 mm; gearing ratio *i* = 2.

Table 1 and Figure 7 present the profile of the gear shaped tool and the coordinates for the profile with one tooth, in the X_1Y_1 reference system.

Table 1. *Coordinates of the flank profile*

Crt. no.	X_1 [mm]	Y_1 [mm]	Crt. no.	X_1 [mm]	Y_1 [mm]
1	30.926	4.488	7	39.050	12.550
2	32.221	5.009	8	39.994	14.564
3	33.781	6.017	9	40.779	16.672
4	35.293	7.342	10	41.405	18.854
5	36.691	8.906	11	41.873	21.092
6	37.947	10.654			

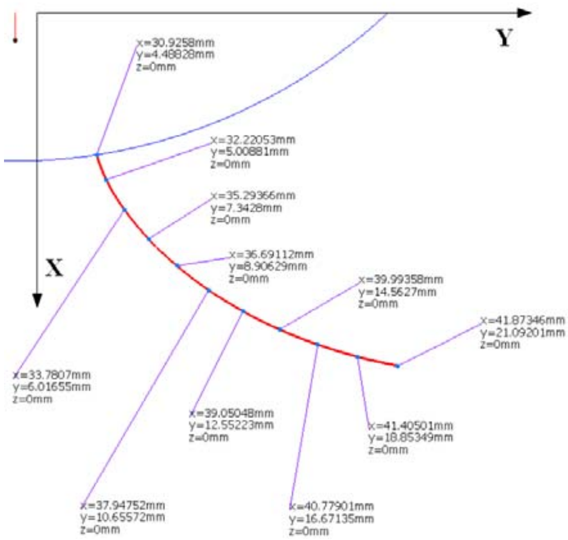

Fig. 7. *Profile of the gear shaped tooth*

4.2. Graphical solution in CATIA

For the above shown profile it was applied the algorithm presented in section 1.

The comparison between the results obtained with the graphical method and those obtained with the analytical one is shown in table 2. These results prove that the points found onto the tool's profile are identical.

The tool's profile is presented in Figure 7.

Table 2. *Comparison between the two methods*

Crt. no.	Analytical method		Graphical method	
	X_1 [mm]	Y_1 [mm]	X_1 [mm]	Y_1 [mm]
1	30.926	4.488	30.926	4.488
2	32.221	5.009	32.221	5.009
3	33.781	6.017	33.781	6.017
4	35.293	7.343	35.294	7.343
5	36.691	8.906	36.691	8.906
6	37.947	10.654	37.947	10.654
7	39.050	12.550	39.050	12.550
8	39.994	14.564	39.994	14.564
9	40.779	16.672	40.779	16.672
10	41.405	18.854	41.405	18.854
11	41.873	21.092	41.873	21.092

5. GEAR SHAPED TOOL FOR A POLYGONAL BUSH

There are technologies for machining holes with squared or hexagonal sections by shaping with gear shaped cutting tool.

The issue solution for gear shaped tool's profiling is proposed, using the relative generating trajectories method (figure 5).

In figure 8, the bush shape is presented together with the crossing section, the pair of rolling centrodes and the reference systems:

- C_1 is the centrode joined with the bush, a circle with radius R_{r1};
- C_2 — centrode joined with the gear shaped tool, a circle with radius R_{r2};
- i — gearing ratio, $i = {\varphi_2}/{\varphi_1} = {R_{r1}}/{R_{r2}}$;
- xy — global reference system;
- XY — mobile reference system, joined with centrode C_1;
- X_1Y_1 — mobile reference system joined with centrode C_2.

The equations of the C_Σ profile are:

$$C_\Sigma \left| \begin{aligned} &X = -a; \\ &Y = u. \end{aligned} \right. \tag{42}$$

The variation limits for parameter u are: $u_{\min} = -a$ and $u_{\max} = a$.

The normal to the C_Σ profile is:

$$\vec{N}_{C_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{i} \, . \tag{43}$$

The in-plane generating trajectories family (27) has equations:

$$\begin{aligned} &X_1 = -a \cdot cos(i-1)\cdot\varphi_1 - u \cdot sin(i-1)\cdot\varphi_1 + \\ &+A_{12}\cdot cos(i\cdot\varphi_1); \\ &Y_1 = -a\cdot sin(i-1)\cdot\varphi_1 + u\cdot cos(i-1)\cdot\varphi_1 + \\ &+A_{12}\cdot sin(i\cdot\varphi_1). \end{aligned} \tag{44}$$

The enveloping condition (31) is:

$$\varphi_1 = arcsin\left({u}/{R_{r1}} \right), \tag{45}$$

with $A_{12} = R_1 - R_{r2}$.

The (44) and (45) equation assemblies represent the profile of the gear shaped tool.

5.1. Numerical application

A numerical application for a squared bush with a = 40 mm and i = 4/3 is presented.

In Table 3 and Figure 9 are presented the coordinates and the form of the gear shaped cutting tool.

Table 3. *Coordinates of the gear shaped tool's profile*

Crt. no.	X_1 [mm]	Y_1 [mm]	Crt. no.	X_1 [mm]	Y_1 [mm]
1	-21.213	-36.742	**7**	-25.687	7.223
2	-22.993	-29.161	**8**	-25.169	14.468
3	-24.280	-21.766	**9**	-24.285	21.766
4	-25.169	-14.468	**10**	-22.993	29.161
5	-25.687	-7.223	**11**	-21.213	36.742
6	-25.858	0			

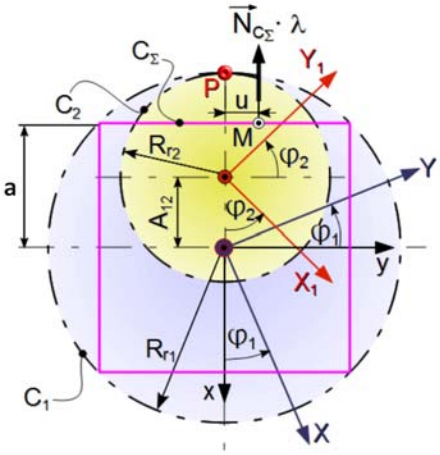

Fig. 8. *Profile of the gear shaped cutting tool*

5.2. Graphical solution in CATIA

The same problem was solved using the capabilities of the CATIA design environment.

The comparative results are presented in table 4 and the tool's profile is shown in Figure 9.

Table 4. *Coordinates of the gear shaped tool's profile obtained by analytical and graphical methods*

Crt. no.	Analytical method		Graphical method	
	X_1 [mm]	Y_1 [mm]	X_1 [mm]	Y_1 [mm]
1	-21.213	-36.742	-21.213	-36.742
2	-22.993	-29.161	-22.993	-29.161
3	-24.280	-21.766	-24.280	-21.766
4	-25.169	-14.468	-25.169	-14.468
5	-25.687	-7.223	-25.687	-7.223
6	-25.858	0	-25.858	0
7	-25.687	7.223	-25.687	7.223
8	-25.169	14.468	-25.169	14.468
9	-24.285	21.766	-24.285	21.766
10	-22.993	29.161	-22.993	29.161
11	-21.213	36.742	-21.213	36.742

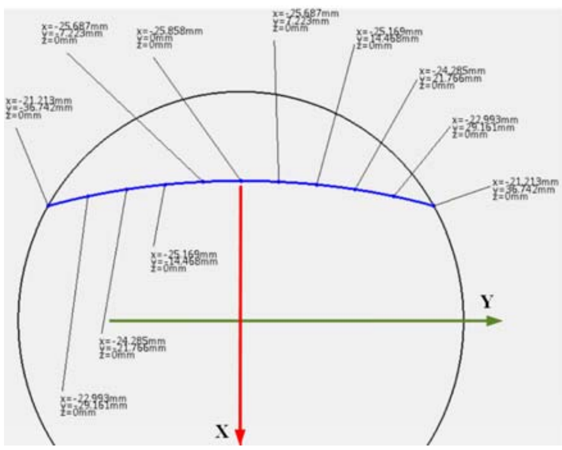

Fig. 9. *Profile of the gear shaped tooth*

6. CONCLUSIONS

This paper proposes a graphical solution for the issue of profiling tools which generate by enwrapping.

The solution is based on the description of the relative trajectories obtained in the generating process. These are the trajectories of the point belonging to the profile to be generated in their motions in the tool's space.

A new form for the enveloping condition is proposed.

The obtained results prove that the solution is simple, easy to apply and rigorous.

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