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GRAPHICAL SOLUTION IN CATIA FOR PROFILING ROTARY CUTTERS. THE METHOD OF RELATIVE TRAJECTORIES

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ABSTRACT

This paper deals with a new form of the enwrapping condition in case of profile enwrapping associated with a pair of rolling centrodes, at generating with rotary cutters. The problem can be solved appealing to the capabilities of the CATIA design environment. The graphical environment is used for profiling the rotary cutter associated with a circular centrode, in rolling with a linear centrode, associated with in-plane profiles representing a cylindrical helical surface with constant pitch in the case of long threads.

A specific theorem is stated and specific algorithms are defined for determining the rotary cutter's profile. Numerical examples are presented and solutions are compared with those obtained by analytical methods.

KEYWORDS: rotary cutter, relative trajectories, enveloping, CATIA

1. INTRODUCTION

The issue of profiling tools which generate by enwrapping, with the method of rolling, as the tools associated with a pair of rolling centrodes (rack-gear tool, rotary cutter or gear shaped tool), is known and has solutions based on the basic theorems of surfaces enveloping: the first theorem of Olivier [1], [2], [6], Gohman theorem [1] or complementary theorems such as the theorem of “minimum distance”, the method of “substitutive circles family”, the method of “in-plane generating trajectories” [8], [9].

Solutions developed in graphical environments are also known. These solutions are developed in AutoCAD [4], [5], or CATIA [10] as applications for the generation of ordered curls of profiles associated with a pair of rolling centrodes in the cases of generating rack-gear, gear shaped tool or rotary cutter.

A new approach to rotary cutter profiling is presented in this paper. The cutter is designed for machining a cylindrical helical surface with constant pitch. The solution is based on knowledge of relative trajectories of points belonging to the axial section of the thread to be generated in the rolling movement of the two associated centrodes: the linear centrode associated with the axial section of the thread and the circular centrode associated with the rotary cutter, in the generation plane.

In this way, the relative trajectories family is determined for points belonging to the profile to be generated. This family is the profile of the rotary cutter. A solution based on the method of relative generating trajectories is presented in [7], determining, in principle, the family of relative trajectories and the enwrapping condition.

A numerical example for generating a thread with trapezoid axial profile was also presented in [7].

In this paper, some algorithms for elaboration are proposed based on the trajectories of points belonging to the profile to be generated relative to the generating tool. The algorithms are developed based on CATIA design environment capabilities. At the same time, the relative trajectories are analytically defined and a specific theorem is stated for the case of the ordered curl of profiles generated with a rotary cutter.

2. GENERATION PRINCIPLE

In figure 1, the assembly of rolling centrodes, the reference systems and the angular parameter of revolution around the rotary cutter axis are presented.

The reference systems are defined as:

- *xy* is the global reference system;
- *XY* — mobile reference system joined with the C_1 centrode, associated with the generated thread;

- X_1Y_1 — mobile reference system associated with the C_2 centrode of the rotary cutter.
- φ — angular parameter of X_1Y_1 reference system rotation;
- λ — movement parameter of XY reference system translation.

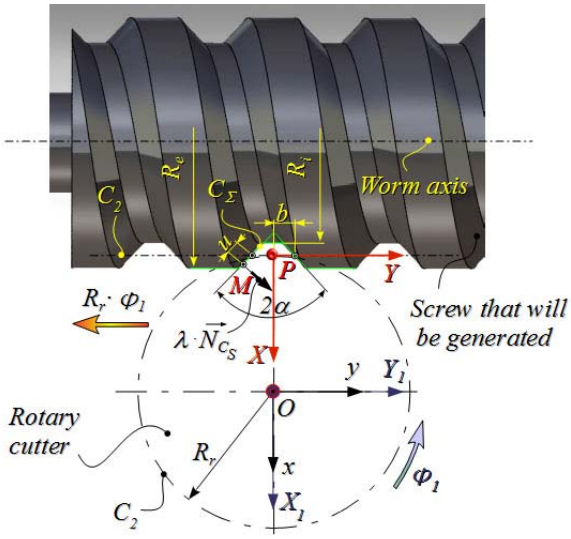

Fig. 1. *Rolling centrodes: C1 - centrode associated with the axial section of the generated thread; C2 - centrode associated with the rotary cutter*

The rolling condition between the C_1 and C_2 centrodes is:

$$\lambda = R_{rs} \cdot \varphi \tag{1}$$

where Rrs is the C2 circular centrode radius.

A segment of the composed profile has general equations if form:

$$\Sigma \left| \begin{aligned} X &= X(u); \\ Y &= Y(u). \end{aligned} \right. \tag{2}$$

where u is the variable parameter.

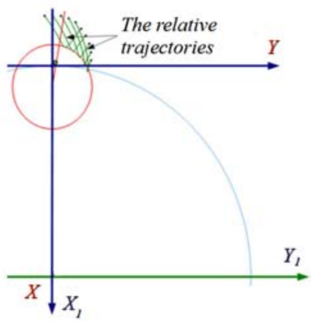

Fig. 2. *Relative trajectory family (cycloid trajectories) and the S envelope — the profile of the rotary cutter*

In the relative motion regarding the tool’s centrode:

$$X_1 = \omega_3(\varphi)\left[X + \begin{pmatrix} -R_{r_s} \\ -R_{r_s} \cdot \varphi \end{pmatrix} \right] \tag{3}$$

the trajectories family is determined for various values of u parameter, in principle, in form:

$$(\Sigma_u)_\varphi \left| \begin{aligned} X_1 &= X_1(u, \varphi); \\ Y_1 &= Y_1(u, \varphi). \end{aligned} \right. \tag{4}$$

The parametrical equations in form (4) represent, in principle, a curves’ family whose enveloping, in reference system joined with C_2 centrode (the X_1Y_1 system), represents the profile of the rotary cutter determined based on the relative trajectories method (see [7]). A specific enwrapping condition for this envelope is determined.

3. ROTARY CUTTER FOR A BALL SCREW

3.1. Analytical solution

The simplest form of a ball screw is presented in Figure 3, as a circular axial profile.

The reference systems are defined according to figures 1 and 2:
- xy is the global reference system;
- XY — mobile reference system, joined with the C_1 centrode;
- X_1Y_1 — reference system joined with the C_2 centrode.

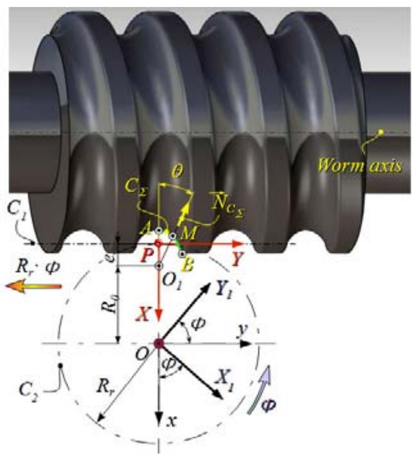

Fig. 3. *Ball screw; axial section; conjugated centrodes; reference systems*

The profile’s equations in the XY reference system:

$$C_\Sigma \left| \begin{aligned} X &= (R_{rs} - R_0) - r \cdot cos\,\theta; \\ Y &= r \cdot sin\,\theta. \end{aligned} \right. \tag{5}$$

The versor of normal to the C_Σ profile:

$$\vec{n}_{C_\Sigma} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r\cdot\sin\theta & r\cdot\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \left[r\cdot\cos\theta\cdot\vec{i} - r\cdot\sin\theta\cdot\vec{j}\right]\frac{1}{r} \quad (6)$$

The definition of the directrix parameters of the vector normal to the C_Σ profile allows writing the normal in the point M,

$$\begin{vmatrix} X = \left(R_{rs} - R_0\right) - r\cdot\cos\theta + \beta\cdot\cos\theta; \\ Y = r\cdot\sin\theta - \beta\cdot\sin\theta. \end{vmatrix} \quad (7)$$

with β scalar parameter and θ variable parameter which, for the AB zone of the circle's arc, varies between

$$\theta_{min} = 0;\ \theta_{max} = arccos\left[\frac{R_{rs} - R_0}{r}\right]. \quad (8)$$

The value $R_{rs} - R_0 = e$ is defined.

The normals in current points of the C_Σ profile (see figure 2) describe, in the relative motion in space X_1Y_1, a lines family:

$$\left(N_{C_\Sigma}\right)_\varphi : \begin{vmatrix} X_1 = \left(e - r\,cos\,\theta + \beta\,cos\,\theta - R_{rs}\right)cos\,\varphi + \\ +\left(r\,sin\,\theta - \beta\,sin\,\theta - R_{rs}\varphi\right)sin\,\varphi; \\ Y_1 = -\left(e - r\,cos\,\theta + \beta\,cos\,\theta - R_{rs}\right)sin\,\varphi + \\ +\left(r\,sin\,\theta - \beta\,sin\,\theta - R_{rs}\varphi\right)cos\,\varphi. \end{vmatrix} \quad (9)$$

The condition that the normals' family (9) (for β-variable) pass through the gearing pole is imposed:

$$P\begin{vmatrix} X_1 = -R_{rs}\,cos\,\varphi \\ Y_1 = R_{rs}\,sin\,\varphi \end{vmatrix} \quad (10)$$

resulting the conditions:

$$\begin{aligned} &\left(e - r\,cos\,\theta + \beta\,cos\,\theta - R_{rs}\right)cos\,\varphi + \\ &+\left(r\,sin\,\theta - \beta\,sin\,\theta - R_{rs}\varphi\right)sin\,\varphi + R_{rs}\,cos\,\varphi = 0; \quad (11) \\ &-\left(e - r\,cos\,\theta + \beta\,cos\,\theta - R_{rs}\right)sin\,\varphi + \\ &+\left(r\,sin\,\theta - \beta\,sin\,\theta - R_{rs}\varphi\right)cos\,\varphi - R_{rs}\,sin\,\varphi = 0. \end{aligned}$$

which, particularized for equations (5), that describe the axial profile of the ball screw and regarding the definitions:

$$\begin{aligned} \dot{X}_\theta &= r\,sin\,\theta; \\ \dot{Y}_\theta &= r\,cos\,\theta, \end{aligned} \quad (12)$$

will determine the enwrapping condition

$$\varphi = \frac{e}{R_{rs}}\cdot tg\theta\,. \quad (13)$$

For β = 0, the equations (11) represent the relative trajectories family of points belongs to the C_Σ profile regarding the X_1Y_1 reference system. The equations of this family are:

$$\left(T_{(\theta)}\right)_\varphi \begin{vmatrix} X_1 = \left(e - r\cdot cos\,\theta - R_{rs}\right)\cdot cos\,\varphi + \\ +\left(r\cdot sin\,\theta - R_{rs}\varphi\right)\cdot sin\,\varphi; \\ Y_1 = -\left(e - r\cdot cos\,\theta - R_{rs}\right)\cdot sin\,\varphi + \\ +\left(r\cdot sin\,\theta - R_{rs}\varphi\right)\cdot cos\,\varphi. \end{vmatrix} \quad (14)$$

The enwrapping of trajectories family (14) represents the profile of the rotary cutter's tooth flank.

The equations of normal to the C_Σ profile are in form:

$$\begin{aligned} X &= X(\theta) + \beta\cdot\dot{Y}_\theta; \\ Y &= Y(\theta) - \beta\cdot\dot{X}_\theta, \end{aligned} \quad (15)$$

with β variable parameter.

The normals' family in the relative movements regarding the $X_1Y_1Z_1$ reference system is given by:

$$\begin{aligned} X_1 &= \left[X(\theta) + \beta\cdot\dot{Y}_\theta - R_r\right]\cdot cos\,\varphi + \\ &+\left[Y(\theta) - \beta\cdot\dot{X}_\theta - R_r\cdot\varphi\right]\cdot sin\,\varphi; \\ Y_1 &= -\left[X(\theta) + \beta\cdot\dot{Y}_\theta - R_r\right]\cdot sin\,\varphi + \\ &+\left[Y(\theta) - \beta\cdot\dot{X}_\theta - R_r\cdot\varphi\right]\cdot cos\,\varphi, \end{aligned} \quad (16)$$

with φ variable angular parameter in the relative motion of the mobile reference system.

The enwrapping condition specifically for the generating relative trajectories method is determined eliminating the β parameter,

$$\varphi = \frac{X(\theta)\cdot\dot{X}_\theta + Y(\theta)\cdot\dot{Y}_\theta}{R_{rs}\cdot\dot{Y}_\theta}, \quad (17)$$

The assembly of equations (14) and (13) represents the enwrapping of the profile (5), C_Σ, namely the searched solution, the profile of the rotary cutter.

3.2. Numerical application

It was considered an example for a profile with dimensions: Rrs = 50 mm; r = 10 mm; R_0 = 45 mm.

The coordinates of rotary cutter's flank are presented in Table 1.

Table 1. *Coordinates of the flank profile*

Crt. no.	X_1 [mm]	Y_1 [mm]
1	-55	0
2	-54.937	1.097
3	-54.748	2.180
4	-54.434	3.236
5	-54.000	4.249
6	-53.449	5.206
7	-52.787	6.093
8	-52.022	6.895
9	-51.165	7.596
10	-50.231	8.177
11	-49.252	8.617

3.3. Graphical application

For the same profile, a graphical application was developed in CATIA design environment. The tool’s profile is presented in Figure 4 and the comparative results are presented in Table 2.

Table 2. *Comparison between the two methods*

Crt. no.	*Analytical method*		*Graphical method*	
	X_1 [mm]	Y_1 [mm]	X_1 [mm]	Y_1 [mm]
1	-55	0	-55	0
2	-54.937	1.097	-54.937	1.097
3	-54.748	2.180	-54.747	2.181
4	-54.434	3.236	-54.434	3.236
5	-54.000	4.249	-54.000	4.249
6	-53.449	5.206	-53.449	5.206
7	-52.787	6.093	-52.787	6.093
8	-52.022	6.895	-52.022	6.895
9	-51.165	7.596	-51.165	7.596
10	-50.231	8.177	-50.231	8.177
11	-49.252	8.617	-49.252	8.617

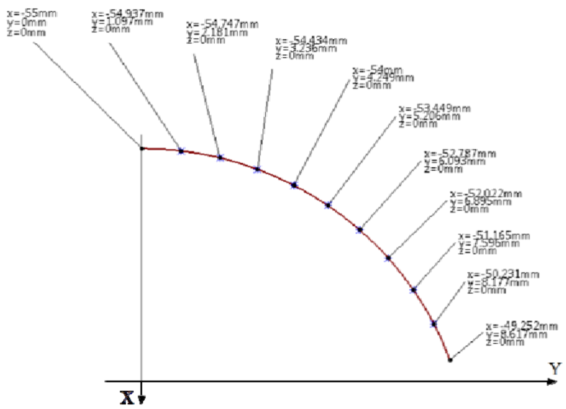

Fig. 4. *Profile of rotary cutter tool*

4. CONCLUSIONS

The generating trajectories method represents a complementary method for the study of profile enwrapping, associated with a pair of rolling centrodes. The graphical method was developed in CATIA and has the advantage to be simple and to offer very rigorous solutions using the capabilities of the graphical design environment.

The comparison between the results obtained analytically and those obtained graphically proves the graphical method’s quality. This method also presents the advantage to give solutions for enveloping problems.

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