THE ANNALS OF "DUNĂREA DE JOS" UNIVERSITY OF GALAȚI
FASCICLE V, TECHNOLOGIES IN MACHINE BUILDING, ISSN 1221-4566
2008

Algorithm for Gear Shaped Tool Profiling by Bezier Approximation

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ABSTRACT

The profiling of tools associated with centrodes, which generated by enwrapping, by rolling method, ordered profiles curls may be done, as in the case of rack-gear tool using the surfaces fundamentals methods.

Multiple situations in the industrial practice don't need a very rigorous in the defining of these profiles.

As follow, the elaboration of a method and adequate algorithms which allow the profiling, in certain precision limits, of the tools, but in simple calculus forms, for various types of elementary profiles, may be useful in industrial appliances.

In this paper, is proposed a method to approximate the gear shaped tool, regarding the representation using the Bezier polynomials of 2nd or 3rd degree.

A small Bezier polynomials degree leads at simple expressions of the polynomials coefficients, allowing a pre-calculus of these.

***Keywords**: Gear shaped tool, cutting l tools, Bezier polynomials.*

1. Introduction

The profiling of tools associated with centrodes, which generated by enwrapping, by rolling method, ordered profiles curls —the case of the gear shaped cutter and rotary cutter, may be done, as in the case of rack-gear tool using the surfaces fundamentals methods: Olivier theorem and Gohman method [2],[4]; as so as the complementary theorems as "the minimum distance method" [7], the "substitutive circles family" method [7], the tangents method [7], the in-plane generating trajectories [7], [10] and solid modelling [3].

Also, the problem may be solved using the solid modelling method [5],[10].

All these methods offer rigorous solutions which allow the very precisely profiling of the gear shaped tool's teeth.

Multiple situations in the industrial practice doesn't need a very rigorous in the defining of these profiles (for example in case on tool's profiling for polygonal bore — square, hexagon or tools for internal flute).

As follow, the elaboration of a method and adequate algorithms which allow the profiling, in certain precision limits, of the tools, but in simple calculus forms, for various types of elementary profiles, may be useful in industrial appliances.

In this paper, is proposed a method to approximate the gear shaped tool, regarding the representation using the Bezier polynomials of 2nd or 3rd degree [1], [8], [9].

A small Bezier polynomials degree leads at simple expressions of the polynomials coefficients, allowing a pre-calculus of these.

The goal is to verify this method by comparing the results obtained by proposed method, for elementary profiles: straight segment, circle arc, involute arc, regarding the classical methods for enveloping generating tools profiling.

2. Reference systems – generation kinematics

They are defined:

- the rolling centrodes couple, C_1 and C_2, of the blank and respectively of the tool;
- *xyz* — the global reference system, with *z* axis, as axis of centrode associated with the profiles curl to be generated;
- *XYZ* — relative reference system, joined with the profile to be generated;

- ξηζ — relative reference system joined with the gear shaped tool.

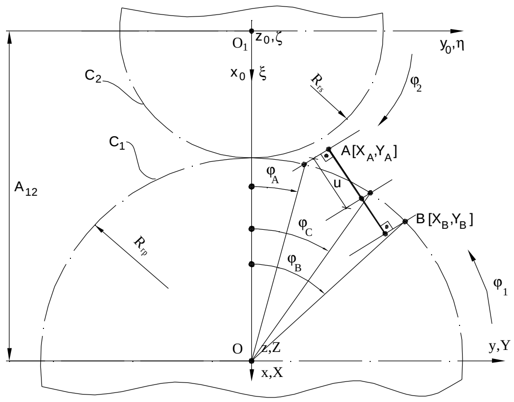

Fig. 1. *Reference systems*

The generating process kinematics presume the rotation movement of the two centrodes, with the respect of the rolling condition:

$$R_{rp} \cdot j_1 = R_{rs} \cdot j_2 . \qquad (1)$$

The absolute movement of the two centrodes are described by transformation:

$$x = w_3^T(j_1) \cdot X , \qquad (2)$$

representing the rotation of the C_1 centrode and of the space joined with these, around the z axis;

$$x_0 = w_3^T(-j_2) \cdot x , \qquad (3)$$

representing the rotation of the tool’s centrode.

The relative position of the global reference systems is give by transformation:

$$x_0 = x + A \quad A = \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix}; A_{12} = R_{rp} + R_{rs} \qquad (4)$$

so, from (2),(3) and (4), may be defined the relative movement:

$$x = w_3(-j_2) \cdot \left[w_3^T(j_1) \cdot X - A \right], \qquad (5)$$

movement, which describe the profiles family (the trajectory family of points belong to the profile to be generated) regarding the gear shaped tool’s reference system.

3. Gear shaped tool profiling algorithm

Is proposed, as profile to be generated, the $\overline{AB}$ straight segment, with equations:

$$\Delta \left| \begin{aligned} X &= X_A + u \cos a ; \\ Y &= Y_A + u \sin a . \end{aligned} \right. \qquad (6)$$

In equation (6) are defined:

- the ends coordinates of the $\overline{AB}$ straight segment,

$$A[X_A, Y_A] \text{ and } B[X_B, Y_B]; \qquad (7)$$

- the a angle,

$$tg a = \frac{|Y_B - Y_A|}{|X_B - X_A|} . \qquad (8)$$

Now, we can determine the j_1 rolling angles values for various points from $\overline{AB}$ profile, see figure 1.

Corresponding to the point *A*, is determined, along the normal in *A* to $\overline{AB}$ profile, the intersection point with the C_1 centrode, with R_{rp} radius, be *A’* this point.

The rolling angle corresponding to the *A’* point is determined with relation

$$j_{1A} = \arccos\left[\frac{X_A \cos a + Y_A \sin a}{R_{rp}} \right] + a . \qquad (9)$$

Similarly may be determined the angles:

$$j_{1B} = \arccos\left[\frac{X_B \cos a + Y_B \sin a}{R_{rp}} \right] + a , \qquad (10)$$

corresponding to *B’* (for the end point *B*) and

$$j_{1C} = \arccos\left[\frac{X_C \cos a + Y_C \sin a}{R_{rp}} \right] + a , \qquad (11)$$

for an intermediate point on the profile (not represented in figure).

The knowledge of these angular values allow to know the coordinated of points belong to the gear shaped profile, in the *xhz* reference system, form (5), in form:

$$\begin{aligned} x &= [X_A + u \cos a] \cos[(1+i) j_1] - \\ &\quad - [Y_A + u \sin a] \sin[(1+i) j_1] + A_{12} \cos(i j_1); \\ h &= [X_A + u \cos a] \sin[(1+i) j_1] + \\ &\quad + [Y_A + u \sin a] \cos[(1+i) j_1] + A_{12} \sin(i j_1), \end{aligned} \qquad (12)$$

for $$j_2 = i \cdot j_1 = \frac{R_{rp}}{R_{rs}} \cdot j_1 , \qquad (13)$$

i — is the gear ratio.

For various values of the “u” parameter:

$u = 0$ – *pentru punctul A*;

$u = 0.5u_{max}, u_{max} = \sqrt{[X_B - X_A]^2 + [Y_B - Y_A]^2}$, ; (14)

pentru punctul C

$u = u_{max}$, *pentru punctul B*,

in table 1, are calculated the Bezier polynomial coefficients, of the gear shaped cutter, reciprocally enveloping with the straight segment profile (6).

They are defined, in table 1, the 2nd degree substitution Bezier polynomial, for approximation of a gear shaped tool, reciprocally enveloping with an ordered profile curl in straight line form of segment $\overline{AB}$:

Similarly, for a 3rd degree approximation Bezier polynomial, on form:

$$\begin{aligned} \xi &= \lambda^3 A_\xi + 3\cdot\lambda^2\cdot(1-\lambda)\cdot_\xi + \\ &+ 3\cdot\lambda\cdot(1-\lambda)^2\cdot C_\xi + (1-\lambda)^3\cdot D_\xi; \\ \eta &= \lambda^3 A_\eta + 3\cdot\lambda^2\cdot(1-\lambda)\cdot B_\eta + \\ &+ 3\cdot\lambda\cdot(1-\lambda)^2\cdot C_\eta + (1-\lambda)^3\cdot D_\eta, \end{aligned} \quad (16)$$

in table 2, are represented the calculus relations of the polynomial coefficients.

Table 1 *Bezier 2nd polynomial coefficients*

	Coordinates of profile to be generated	Rolling angle
0	X_A, Y_A	$\varphi_A = \arccos\left[\frac{X_A\cos\alpha + Y_A\sin\alpha}{R_{rp}}\right] + \alpha$
0.5 u_{max}	$X_C = 0.5\cdot X_A + 0.5\cdot X_B$ $Y_C = 0.5\cdot Y_A + 0.5\cdot Y_B$	$\varphi_C = \arccos\left[\frac{X_C\cos\alpha + Y_C\sin\alpha}{R_{rp}}\right] + \alpha$
u_{max}	X_B, Y_B	$\varphi_B = \arccos\left[\frac{X_B\cos\alpha + Y_B\sin\alpha}{R_{rp}}\right] + \alpha$
λ	Points on rack-gear profile	Approximation polynomial coefficients
1	$\xi_A = X_A\cos[(1+i)\varphi_{1A}] -$ $-Y_A\sin[(1+i)\varphi_{1A}] + A_{12}\cos(i\varphi_{1A})$ $\eta_A = X_A\sin[(1+i)\varphi_{1A}] +$ $+Y_A\cos[(1+i)\varphi_{1A}] + A_{12}\sin(i\varphi_{1A})$	$A_\xi = \xi_A$ $A_\eta = \eta_A$
0.5	$\xi_C = X_C\cos[(1+i)\varphi_{1C}] -$ $-Y_C\sin[(1+i)\varphi_{1C}] + A_{12}\cos(i\varphi_{1C})$ $\eta_C = X_C\sin[(1+i)\varphi_{1C}] +$ $+Y_C\cos[(1+i)\varphi_{1C}] + A_{12}\sin(i\varphi_{1C})$	$C_\xi = \frac{\xi_C - 0.25\cdot\xi_A - 0.25\cdot\xi_B}{0.5}$ $C_\eta = \frac{\eta_C - 0.25\cdot\eta_A - 0.25\cdot\eta_B}{0.5}$
0	$\xi_B = X_B\cos[(1+i)\varphi_{1B}] -$ $-Y_B\sin[(1+i)\varphi_{1B}] + A_{12}\cos(i\varphi_{1B})$ $\eta_B = X_B\sin[(1+i)\varphi_{1B}] +$ $+Y_B\cos[(1+i)\varphi_{1B}] + A_{12}\sin(i\varphi_{1B})$	$B_\xi = \xi_B$ $B_\eta = \eta_B$

$$\begin{aligned} \xi &= \lambda^2 A_\xi + 2\cdot(1-\lambda)\cdot\lambda\cdot C_\xi + (1-\lambda)^2\cdot B_\xi; \\ \eta &= \lambda^2 A_\eta + 2\cdot(1-\lambda)\cdot\lambda\cdot C_\eta + (1-\lambda)^2\cdot B_\eta. \end{aligned} \quad (15)$$

4. Analytical profiling method

Is proposed the comparing of results for the profiling algorithm by Bezier polynomial approximation, with results obtained for the same profile, by a rigorous analytical method — the in-plane generating trajectories [8], [10].

According to this method, the gear shaped tool's profile may be obtained associating with the equations (12), the enveloping condition

Table 2 *Bezier 3rd polynomial coefficients*

	Coordinates of profile to be generated	Rolling angle
0	X_A, Y_A	$j_A = \arccos\left[\frac{X_A \cos a + Y_A \sin a}{R_{rp}}\right] + a$
1/3 u_{max}	$X_C = X_A + \left(\frac{1}{3}\right)[X_B - X_A]$ $Y_C = Y_A + \left(\frac{1}{3}\right)[Y_B - Y_A]$	$j_C = \arccos\left[\frac{X_C \cos a + Y_C \sin a}{R_{rp}}\right] + a$
2/3 u_{max}	$X_D = X_A + \left(\frac{2}{3}\right)[X_B - X_A]$ $Y_D = Y_A + \left(\frac{2}{3}\right)[Y_B - Y_A]$	$j_D = \arccos\left[\frac{X_D \cos a + Y_D \sin a}{R_{rp}}\right] + a$
u_{max}	X_B, Y_B	$j_B = \arccos\left[\frac{X_B \cos a + Y_B \sin a}{R_{rp}}\right] + a$
λ	Points on rack-gear tool’s profile	Approximation polynom coefficients
1	$x_A = X_A \cos\left[(1+i) j_{1A}\right] -$ $-Y_A \sin\left[(1+i) j_{1A}\right] + A_{12} \cos(i j_{1A})$ $h_A = X_A \sin\left[(1+i) j_{1A}\right] +$ $+Y_A \cos\left[(1+i) j_{1A}\right] + A_{12} \sin(i j_{1A})$	$D_x = x_A$ $D_h = h_A$
1/3	$x_C = X_C \cos\left[(1+i) j_{1C}\right] -$ $-Y_C \sin\left[(1+i) j_{1C}\right] + A_{12} \cos(i j_{1C})$ $h_C = X_C \sin\left[(1+i) j_{1C}\right] +$ $+Y_C \cos\left[(1+i) j_{1C}\right] + A_{12} \sin(i j_{1C})$	$C_x = \frac{18 \cdot x_C - 9 \cdot x_B + 2 \cdot x_A - 5 \cdot x_D}{6}$ $C_h = \frac{18 \cdot h_C - 9 \cdot h_B + 2 \cdot h_A - 5 \cdot h_D}{6}$
2/3	$x_D = X_D \cos\left[(1+i) j_{1D}\right] -$ $-Y_D \sin\left[(1+i) j_{1D}\right] + A_{12} \cos(i j_{1D})$ $h_D = X_D \sin\left[(1+i) j_{1D}\right] +$ $+Y_D \cos\left[(1+i) j_{1D}\right] + A_{12} \sin(i j_{1D})$	$B_x = \frac{-5 \cdot x_A + 2 \cdot x_D + 18 \cdot x_B - 9 \cdot x_C}{6}$ $B_h = \frac{-5 \cdot h_A + 2 \cdot h_D + 18 \cdot h_B - 9 \cdot h_C}{6}$
0	$x_B = X_B \cos\left[(1+i) j_{1B}\right] -$ $-Y_B \sin\left[(1+i) j_{1B}\right] + A_{12} \cos(i j_{1B})$ $h_B = X_B \sin\left[(1+i) j_{1B}\right] +$ $+Y_B \cos\left[(1+i) j_{1B}\right] + A_{12} \sin(i j_{1B})$	$A_x = x_D$ $A_h = h_D$

$$\frac{x'_{j1}}{x'_u} = \frac{h'_{j1}}{h'_u} \qquad (17)$$

where, $x'_{j1}, h'_{j1}, x'_u, h'_u$ are partial derivative of the equations, regarding the values j_1 and u.

5. Numerical results

In figure 2 and tables 3 and 4, are presented the numerical results regarding the gear shaped tool for a straight segment, with notations:

- ends segments coordinates, $A[X_A, Y_A]$ and $B[X_D, Y_D]$;
- the rolling radius value R_{rp};
- gear ratio, i;
- error regarding the profile determined based on the analytical method of in-plane generating trajectories, measured normal at the theoretically profile - *Err*.
- approximation with a 2nd degree polynomial, table 3;
- approximation with a 3rd degree polynomial, table 4;
- rolling angle value - j.

a) External gear shaped cutter
End coordinates: A[-100,0]; B[-80, 20];
R_{rp} = 100mm

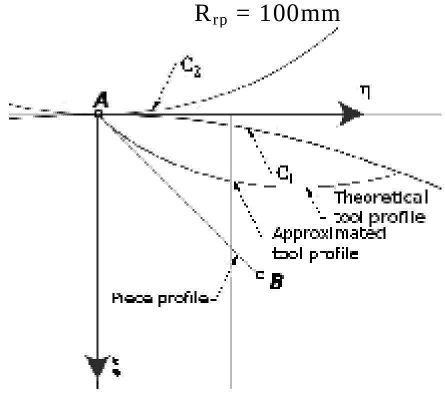

Fig. 2. *Straight segment – external gear shaped cutter*

b) Internal gear shaped cutter
End coordinates: A[-100,0]; B[-80, 20];
R_{rp} = 100mm

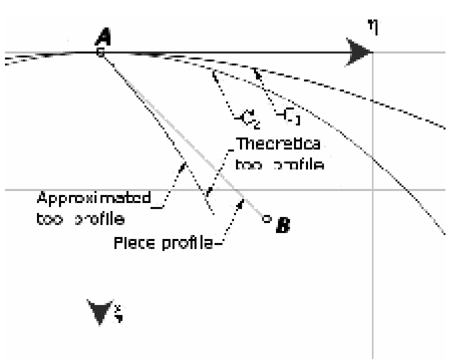

Fig. 3. *Straight segment – internal gear shaped cutter*

6. Conclusion

1. The profiling method for gear shaped tool, generating the profiles curl composed by straight line segments, are rigorous enough, even at approximation using Bezier polynomial with inferior degree.
2. The tabulate algorithm is simple, the approximating polynomial coefficients being pre-calculated.
3. It was used an original software, programmed in Java language, for the results presentation regarding a rigorous profiling method, accepted as comparing element.
4. The increase of approximation polynomial degree leads to the substantial decreasing of the profiling error.

Acknowledgement: This paper was developed under the research programs IDEI, grants ID_656, funded by the Romanian Ministry of Education and Research.

Table 3 *Bezier 2nd polynomial approximation*

λ	Approx ξ [mm]	Approx η [mm]	Tool profile ξ [mm]	Tool profile η [mm]	Err. [mm]	φ [rad]
0.0	**0.00**	**0.00**	**0.00**	**0.00**	**0.000**	**0.000**
0.05	1.01	1.02	0.99	1.05	0.041	0.019
0.1	2.02	2.19	1.98	2.23	0.057	0.039
0.15	2.98	3.48	2.93	3.52	0.059	0.058
0.2	3.90	4.91	3.85	4.92	0.050	0.077
0.25	4.76	6.41	4.73	6.42	0.032	0.095
0.3	5.55	8.02	5.54	8.02	0.012	0.113
0.33	**6.05**	**9.13**	**6.05**	**9.13**	**0.000**	**0.125**
0.35	6.29	9.72	6.29	9.72	0.005	0.131
0.4	6.95	11.50	6.97	11.49	0.020	0.149
0.45	7.54	13.36	7.57	13.35	0.029	0.166
0.5	8.04	15.29	8.08	15.28	0.032	0.184
0.55	8.47	17.28	8.50	17.28	0.028	0.201
0.6	8.80	19.34	8.82	19.33	0.019	0.218
0.65	9.04	21.44	9.04	21.44	0.005	0.234
0.66	**9.09**	**22.12**	**9.09**	**22.12**	**0.000**	**0.240**
0.7	9.18	23.59	9.17	23.59	0.011	0.251
0.75	9.21	25.77	9.18	25.78	0.028	0.267
0.8	9.13	27.99	9.09	27.99	0.043	0.283
0.85	8.94	30.23	8.89	30.23	0.051	0.299
0.9	8.62	32.49	8.57	32.49	0.050	0.315
0.95	8.18	34.75	8.15	34.76	0.034	0.331
1.0	**7.61**	**37.03**	**7.61**	**37.03**	**0.000**	**0.347**

Table 4 *Bezier 3rd polynomial approximation*

λ	Approx ξ [mm]	Approx η [mm]	Tool profile ξ [mm]	Tool profile η [mm]	Err. [mm]	φ [rad]
0.0	**0.00**	**0.00**	**0.00**	**0.00**	**0.000**	**0.000**
0.05	1.00	0.97	1.00	0.98	0.005	0.019
0.1	2.00	1.91	2.00	1.92	0.007	0.039
0.15	3.00	2.82	3.00	2.82	0.007	0.058
0.2	4.00	3.69	4.00	3.70	0.006	0.077
0.25	5.00	4.53	5.00	4.54	0.003	0.095
0.3	6.00	5.35	6.00	5.35	0.001	0.113
0.33	**6.66**	**5.87**	**6.66**	**5.87**	**0.000**	**0.125**
0.35	7.00	6.13	7.00	6.13	0.001	0.131
0.4	8.00	6.88	8.00	6.88	0.002	0.149
0.45	9.00	7.61	9.00	7.61	0.003	0.166
0.5	10.00	8.31	10.00	8.30	0.003	0.184
0.55	11.00	8.98	11.00	8.98	0.003	0.201
0.6	12.00	9.63	12.00	9.63	0.002	0.218
0.65	13.00	10.25	13.00	10.25	0.001	0.234
0.66	**13.32**	**10.45**	**13.32**	**10.45**	**0.000**	**0.240**
0.7	14.00	10.85	14.00	10.86	0.001	0.251
0.75	15.00	11.43	15.00	11.44	0.003	0.267
0.8	16.00	11.99	16.00	12.00	0.004	0.283
0.85	17.00	12.53	17.00	12.53	0.005	0.299
0.9	18.00	13.05	18.00	13.05	0.005	0.315
0.95	19.00	13.54	19.00	13.55	0.003	0.331
1.0	**20.00**	**14.03**	**20.00**	**14.03**	**0.000**	**0.347**

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L'algorithme de profilage par approximation Bezier pour le couteaux roue

Resumé

En cette ouvrage, on propose une méthode pour profilage de couteaux roue, en utilisant les polynômes Bezier du 2-ème ou 3-ème degré. Un petit degré des polynômes Bezier mène aux expressions simples des coefficients des polynômes, permettant un pré calcul de ces derniers.

Algoritm pentru profilarea prin aproximare cu polinoame Bezier a sculelor de tip roată

Rezumat

Profilarea sculelor asociate unor centroide care generează prin înfăşurare, prin metoda rulării, vârtejuri ordonate de profiluri – cazul cuțitului - roată şi a cuțitului rotativ – poate fi făcută, ca şi în cazul cremalierei prin utilizarea metodelor fundamentale ale înfăşurării suprafețelor: teorema OLIVIER şi metoda GOHMAN; precum şi teoremele complementare ca: “metoda distanței minime”; metoda “familiei de cercuri substitutive”, metoda tangentelor, metoda traiectoriilor plane de generare. În lucrare, se propune o metodologie de aproximare a profilurilor sculelor de tip roată, având în vedere o reprezentare a acestora în forma unor polinoame Bezier, de gradul 2 sau 3. Un grad mic al polinoamelor Bezier conduce la exprimări simple a coeficienților polinomului, permițând o formă de precalculare a acestora.

Se urmăreşte verificarea acestei metode prin compararea rezultatelor furnizate prin aplicarea metodologiei propuse, pentru profilul elementar, segment de dreaptă, în raport cu metodele clasice de profilare a sculelor generatoare prin înfăşurare.