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On the Implementation of an Algorithm Improving Approximated Tool Profiles Generating Helical Surfaces

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ABSTRACT

A software application, based on an algorithm to find Bezier polynomial approximations of tools generating various helical surfaces, is presented.

The software was designed as an applet (easily embedded into web pages) and developed using Sun Java Development Kit.

Keywords: *software description, representation by poles, helical milling.*

1. Introduction

Helical surfaces are produces on milling machines by rotating the work-piece while at the same time translating it in the direction of the axis of rotation.

Depending on the helical surface to be obtained we can consider a spatial curve – the *generating profile* – simultaneously translate an rotate this profile, and applying corresponding enveloping condition, we can compute the axial profile of the tool generating the considered helical surface. Moreover, as proven by multiple experiments, enveloping condition can be evaluated on a small number of points on the *generating profile*, and by successive Bezier interpolations, an approximation of the axial tool profile can be easily calculated.

2. The algorithm

Consider P a Bezier polynomial approximation of the *generating profile* (fig.1.a):

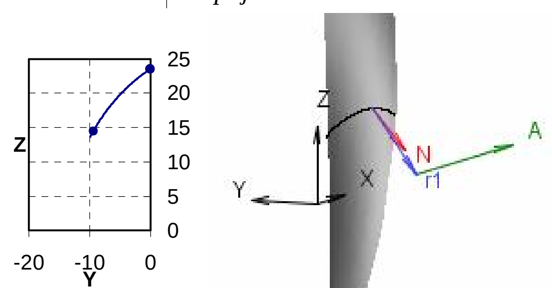
$$P:\begin{vmatrix} X = P_X(l) \\ Y = P_Y(l) \\ Z = P_Z(l) \end{vmatrix},\ l \in [0,1]. \qquad (1)$$

By simultaneously rotating (angle j) and translating P (with pj , where p is a constant)

$$\begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} = \begin{Vmatrix} \cos j & -\sin j & 0 \\ \sin j & \cos j & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} P_X(l) \\ P_Y(l) \\ P_Z(l) \end{Vmatrix} + \begin{Vmatrix} 0 \\ 0 \\ pj \end{Vmatrix}, \qquad (2)$$

the helical surface Π (fig.1.b) is obtained:

$$\Pi(l,j)\begin{vmatrix} X = P_X(l)\cdot\cos j - P_Y(l)\cdot\sin j; \\ Y = P_X(l)\cdot\sin j + P_Y(l)\cdot\cos j; \\ Z = p\cdot j. \end{vmatrix} \qquad (3)$$



(a) **(b)**

Fig. 1. *(a) Circle arc generator – axial plane zOy and (b) corresponding helical surface*

The normal vector for the surface Π can now be easily determinate:

$$\vec{N}_\Pi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \Pi_X'(j) & \Pi_Y'(j) & p \\ \Pi_X'(l) & \Pi_Y'(l) & 0 \end{vmatrix}, \qquad (4)$$

$$\text{or } \vec{N}_\Pi = N_X\vec{i} + N_Y\vec{j} + N_Z\vec{k}. \qquad (5)$$

Depending on the tool used to generate the surface, a different vector $\vec{A}$ should be considered as the tool’s axis.

We can determinate the pairs of parameters (l,j) such that vectors $\vec{N}_\Pi$ and $\vec{A}$ are in the same plane (enveloping condition) by considering a small fixed numbers of values for

l – let $l \in \left(0, \frac{1}{3}, \frac{2}{3}, 1\right)$ – and determination corresponding values for parameter j.

Therefore, four points from the *characteristic curve* (fig. 1.b) are obtained:

$$C_{\Pi} = \begin{Vmatrix} X_{C_{\Pi}, l=0} & Y_{C_{\Pi}, l=0} & Z_{C_{\Pi}, l=0} \\ X_{C_{\Pi}, l=1/3} & Y_{C_{\Pi}, l=1/3} & Z_{C_{\Pi}, l=1/3} \\ X_{C_{\Pi}, l=2/3} & Y_{C_{\Pi}, l=2/3} & Z_{C_{\Pi}, l=2/3} \\ X_{C_{\Pi}, l=1} & Y_{C_{\Pi}, l=1} & Z_{C_{\Pi}, l=1} \end{Vmatrix}. \quad (6)$$

Transforming coordinates from the discrete curve C_{Π}, from helical surface reference system into the tool reference system(fig. 2), another discrete curve with four points, I_{Π} from the surface of the tool can be obtained:

$$I_{\Pi} = \begin{Vmatrix} X_{I_{\Pi}, l=0} & Y_{I_{\Pi}, l=0} & Z_{I_{\Pi}, l=0} \\ X_{I_{\Pi}, l=1/3} & Y_{I_{\Pi}, l=1/3} & Z_{I_{\Pi}, l=1/3} \\ X_{I_{\Pi}, l=2/3} & Y_{I_{\Pi}, l=2/3} & Z_{I_{\Pi}, l=2/3} \\ X_{I_{\Pi}, l=1} & Y_{I_{\Pi}, l=1} & Z_{I_{\Pi}, l=1} \end{Vmatrix}. \quad (7)$$

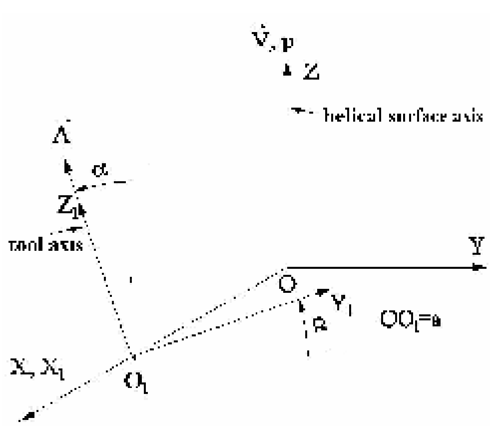


Fig. 2. *System of reference (a global system XYZ and tool system X_I, Y_I, Z_I*

Axial section (fig. 3) for the tool, S_A, can be computed as follows:

$$S_A \begin{vmatrix} H_l = Z_{I_{\Pi,l}} \\ R_l = \left(\sqrt{X_{I_{\Pi,l}}^{\ 2} + Y_{I_{\Pi,l}}^{\ 2}}\right)_i, \end{vmatrix} \quad (8)$$

$$l \in \left(0, \frac{1}{3}, \frac{2}{3}, 1\right)$$

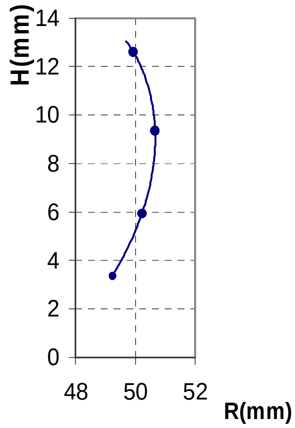


Fig. 3. *Axial disk-tool profile-S_A*

Again, using a polynomial Bezier interpolation of the four points computed as above, from the axial section of the tool, an approximated profile Q for the axial section of the tool can be obtained.

Remark 1.1.

a) If the generating profile is given as a continuous and derivative spatial curve, instead of using a polynomial approximation, the curve itself can be used in the above equations and, considering dense values for the parameter l (e.g.: 10^3 values for l) a precise axial tool profile can be obtained. Those results were used to compare the results of the approximation algorithm.

b) Depending on the tool, $\vec{A}$ - tool's axis, and O_1 - tool origin, should be considered as follow:

- disk tool

$$\vec{A} = -\sin(a)\vec{j} + \cos(a)\vec{k},$$

$$O_1 = (a, 0, 0);$$

- planning tool

$$\vec{A} = -\sin(a)\vec{j} + \cos(a)\vec{k},$$

$$O_1 = (\infty, 0, 0);$$

- end mill tool

$$\vec{A} = \vec{i},$$

$$O_1 = (0, 0, 0).$$

Therefore, coordinates transformation used to obtains S_A can be expressed as follows:

$$\begin{Vmatrix} X_{I_{\Pi}} \\ Y_{I_{\Pi}} \\ Y_{I_{\Pi}} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) \\ 0 & \sin(a) & \cos(a) \end{Vmatrix} \cdot \left(\begin{Vmatrix} X_{G_{\Pi}} \\ Y_{G_{\Pi}} \\ Y_{G_{\Pi}} \end{Vmatrix} - \begin{Vmatrix} O_1 x \\ O_1 y \\ O_1 z \end{Vmatrix} \right)$$

where: $a = 0$ in the case of end mill tool and

$a = \arctan\left(\frac{D}{2p}\right)$ in other cases, and D is the helix outer diameter.

3. Implementation

The main goal of the application is to allow the user to determinate approximated profiles for various tools generating helical surfaces, when a small number of points is known (measured with a 3D measurement machine) for the generating profile.

Also, the algorithm can be used to calculate the approximation error (Remark 1.1, a) for a series analytical profiles: circle arc (axial plane zOy, axial plane zOx, transversal plane xOy), involute generating profile (transversal plane xOy), straight line generating profile (axial plane zOy).

As presented in the Section 2, most computations can be performed without knowing which type of elementary profile (straight line, circle arc or involute) is used as a helix generating profile and the tool is used. If one can provide a method to determine the derivative at any arbitrary position of the generating profile, it is irrelevant from the approximation algorithm's point of view, which type of helical surface is to be generated. Moreover, it suffice to specify the tool's origin and the direction of the tool's axis to transform the coordinates from helical surface coordinate system into the tool's coordinate system, and therefore compute the axial profile for the tool.

This approach fits perfectly the Object Oriented Programming (OOP) paradigm [1], [2]. The application can be easily extended, considering any other elementary profile as helix generator, and just implementing a method to calculate the derivative at any arbitrary position.

4. Application description

In this section, application user interface will be described. The most important visual elements of the application are presented in figure 4 as follows:

1 – select the type of helix generator profile;

2 – configure various parameters of the generating profile (in the case of "Measured points" a list of measured coordinates should be inserted);

3– select the tool type;

4 – configure a series of helix parameters: outer diameter, inner diameter, and the pitch;

5 – update the helical surface displayed on the screen according to parameters and options selected above;

6 – estimate approximation errors;

7 – display tabular comparative results for theoretical profile and approximated tool profile;

8 - characteristic curve on helical surface;

9 - the helical surface;

10 - helical surface origin and coordinates system;

11 - tool's axis and origin of coordinates system.

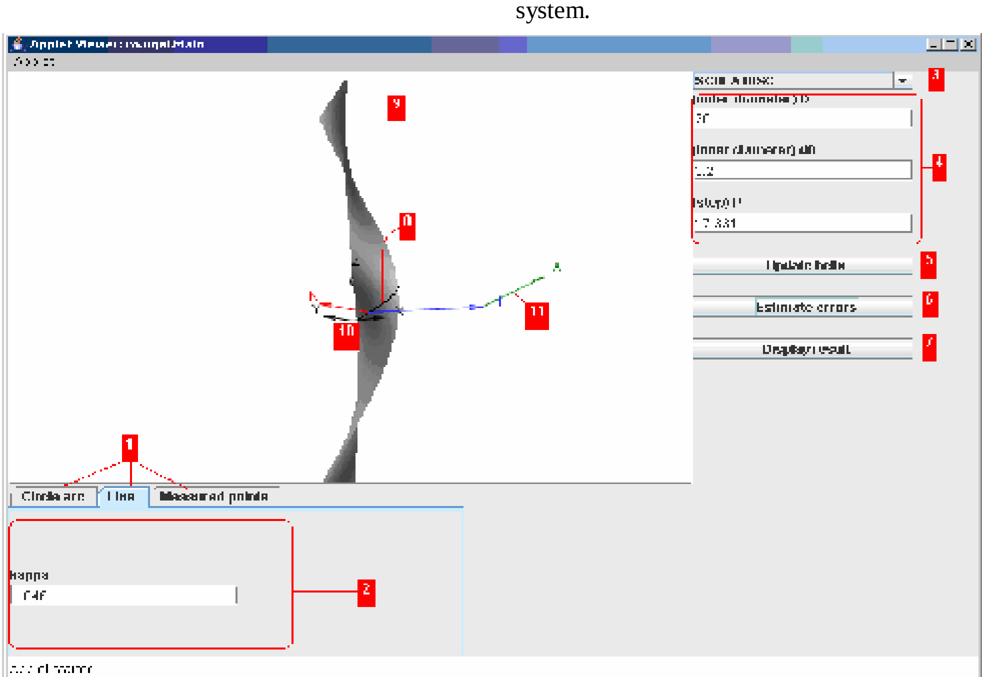


Fig.4. *Application user interface*

5. Conclusions

The software is realized by object oriented programming and is presented as applet. This software is designed to profile tools generating measured helical surfaces.

The numerical examples and the graphical representations are conclusive and easy to use for profile tools disk-tools, cylindrical or frontal-cylinder tools.

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Asupra unui algoritm îmbunătățit pentru determinarea profilurilor sculelor utilizate la prelucrarea suprafețelor elicoidale

Rezumat

În lucrare, este descrisă o aplicație software ce poate fi folosită pentru determinarea unor aproximări eficiente prin polinoame Bezier a profilurilor sculelor generatoare de suprafețe elicoidale: scula disc, scula cilindro-frontala, scula cilindrica. Aplicația a fost construită utilizând *Sun Java Development Kit*, ca un applet uşor de introdus în paginile web.

Sur l'exécution d'un algorithme améliorent des profils rapprochés d'outil produisent des surfaces hélicoïdales

Résumé

Dans ce papier, il est présente un logiciel, basé sur un algorithme spécialisé sur la recherche de l'approximation Bézier de divers outils de génération des surfaces hélicoïdales. Le logiciel a été réalise par un applet (facilement intégrées dans des pages web) et développées à l'aide de Sun Java Development Kit.