DESIGN AND GENERATION OF NONCIRCULAR GEARS WITH CONVEX-CONCAVE PITCH CURVES

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ABSTRACT

The paper introduces a general procedure for noncircular gear design and generation, based on Gielis' supershape,as the gear pitch curve, and on the simulation of the gear cutting process, using rolling method. The procedure algorithm consists of the supershape geometry analysis, to identify and separate convex curves from convex-concave ones, the selection of the proper tools reccomended for "gear cutting process", the limitation of the pitch curve geometry variation, in order to avoid undercutting, and the simulation of rolling. As convex noncircular gear generation was developed in a previous paper, the study is now focused on the convex-concave gear virtual cutting, using a standard shaper cutter and the process' specific kinematics.

KEYWORDS:non-circular gears, convex-concave pitch curve, supershape, shaper.

1. INTRODUCTION

Noncircular gears keep challengingthe scientists due to their ability of producing complex variable speed movements in a simple, compact and reliable way [1]. A curiosity for the old gear industry, with complex geometry and dificulties in manufacture, noncircular gears found limited industrial applications. Nowadays, the facilities offered by virtual modelling and simulation softwares and advanced CNC machine tools, design and manufacture of noncircular gears have become more feasible [2] and encourage the expansion of their applications [3], targeting possible substitutions of cams, linkages, Geneva mechanisms, electrical servomotors etc.

Once thegear pitch curve is defined, an importantfurther step is the generation of the tooth flanks profiles. In this context, scientists used different approaches, such as enveloping theory, analytical generation or manufacturingsimulation.

Danieli [4] generated noncircular gears' tooth profiles, with constant pressure angle, by integrating a differential equation that described the rolling between the tool and the gear. To improve contact between teeth, Danieli and Mundo [5] used a different approach, by mentaining the pressure angle constant for any given tooth, but variable from one tooth to another. Gao et al. [6] divided the pitch curves of elliptic gears into segments and for each segment they used the local curvature radii to generate curvature circles. Based on these circles, they generated the corresponding tooth profiles.

The enveloping theory, introduced by Litvin [7], allowed the generation of noncircular gears' tooth profiles with the same tools used in standard gear cutting process. Based on this theory, Litvin et al. [8] generated elliptical gears with straight and hellical teeth. The same theory was used by Chang and Tsay [9], to generate tooth profile with a shaper cutter, and by Bair [10], to generate circular arc elliptical gears with a rack cutter and a shaper. Based on the geometry principles for spherical engagement, Xia et al. [11] obtained tooth profiles of bevel noncircular gears.

Instead of deducting and solving complicated meshing equations, Li et al. [12] generated noncircular gears' tooth profiles by simulating the cutting process by shaper. The tooth profiles were obtained from the intersection of the shaper profile with the pitch curve's isometric family of curves.

In this paper, the generation of the noncircular gear is based on the following steps:
i) modelling of the noncircular gear pitch curve, byGielis'supershape formula [13]. The supershape is a highly versatile curve, defined by six parameters – the length of the traditional ellipse semi-axes, a multiplication factor of the variable polar angle and three exponents –, whose variations lead to a wide range of shapes for the gear pitch curves and thus, multiple transmission ratio's variation laws in a gear train;

ii) choisen of proper values for the supershape defining parameters, considering the curvature of the pitch curve and the avoidance of pointed vertex;
iii) identification of pitch curve convexity and proper selection of tools. Once the tool is defined, the avoidance of undercutting is considered for the further analysis of pitch curve geometry;
iv) simulation of noncircular gear cutting process.

2. PITCH CURVE GENERATION

In order to generate a pair of conjugated noncircular pitch curves, two main hypothesis can be considered: the definition of the desired transmission function and the definition of the desired driving pitch curve geometry. The approach of pre-designed pitch curve geometry, also known as Generating Profile Method, uses the equation of tranditional or modified ellipse [14, 15], Fourier series [16] or various specific monotonically increasing functions [17].

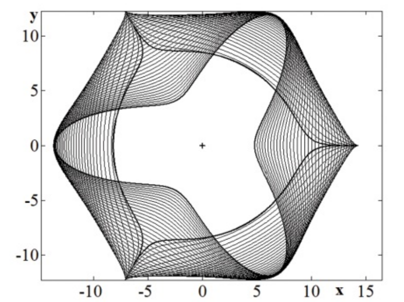
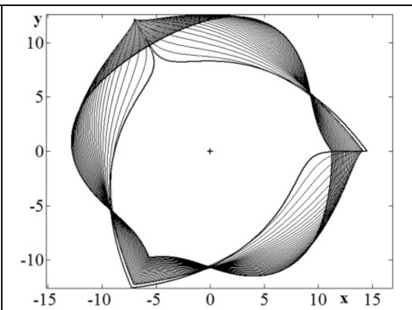
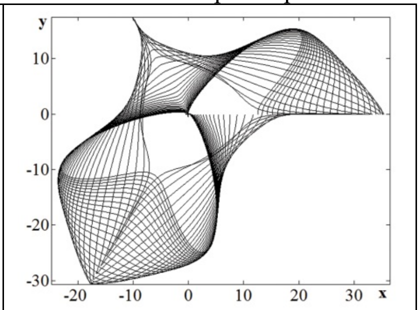
In the attempt of generalysing the generation process of noncircular gears, the authors introduce the Gielis supershape equation to model the driving pitch curve:

$$r(\theta) = \left[\left|\frac{1}{a}\cdot\cos\frac{n\theta}{4}\right|^{n_2} + \left|\frac{1}{b}\cdot\sin\frac{n\theta}{4}\right|^{n_3}\right]^{-\frac{1}{n_1}} \qquad (1)$$

where a, b are nonzero real numbers that define the semi-lengths of the classical ellipse; n - a real number that multiplies the polar angle and defines the number of lobes of the supershape, ie its rotational simmetry; n_1, n_2 and n_3 - real nonzero numbers that lead to pinched, bloated or polygonal, symmetric or asymetric shapes, depending on their values and relationship.

By varying the six defining parameters of the supershape, a wide range of planar curves can be obtained (Tab. 1), but it is obvious that not all of them can be used as gear pitch curves. The appropriate selection of the supershape geometry, admitted as pitch curve for noncircular gears, is based on the limitation of the defining parameters' variation, so that pointed shapes and those with very small curvature radii can be avoided.

Table 1. Supershape families

		
$n = 3$, $a = b = 1$, $n_1 = 1$, $n_2 = n_3 \in [0.2,5]$	$n = 3$, $a = b = 1$, $n_1 = 1$, $n_2 \in [0.2,5]$, $n_3 = 1$	$n = 3$, $a = 1.5$, $b = 1$, $n_1 = 1$, $n_2 = n_3 \in [0.2,5]$

Considering the supershape as a potential gear pitch curve, the exponentsinfluence on the dimensional homogeneity of eq. (1) is excluded by parametrization, using the following notations: $\bar{a} = a/m$, şi $\bar{b} = b/m$, where m is the gear modulus.

As a result, eq (1) can be written as:

$$\bar{r}(\theta) = \left[\left|\frac{1}{\bar{a}}\cdot\cos\frac{n\theta}{4}\right|^{n_2} + \left|\frac{1}{\bar{b}}\cdot\sin\frac{n\theta}{4}\right|^{n_3}\right]^{-\frac{1}{n_1}} \qquad (2)$$

The problem of selecting proper parameters, in order to avoid pointed shapes and those with very small curvature radii, has been covered in a previous paper [18].

3. IDENTIFICATION OF THE CONVEX-CONCAVE PITCH CURVES

Noncircular gears' pitch curves, as supershapes, can be convex or convex-concave curves, based on the defining parameters' values. Considering the necesity to generate tooth profiles, in proper conditions, the curvature radius of the supershape is a priority in controlling undercutting. From the mathematical expression of a planar curve's curvature radius, defined in polar coordinates:

$$\rho(\theta) = \frac{\left[r^2(\theta) + \left(\frac{dr(\theta)}{d\theta}\right)^2\right]^{\frac{3}{2}}}{r^2(\theta) + 2\left(\frac{dr(\theta)}{d\theta}\right)^2 - r(\theta)\frac{d^2r(\theta)}{d\theta^2}} \qquad (3)$$

it results that convex curves are generated as long as the curvature radius remains positive, respectively, the function's denominator:

$$n_\rho(\theta) = r^2(\theta) + 2\left(\frac{dr(\theta)}{d\theta}\right)^2 - r(\theta)\frac{d^2r(\theta)}{d\theta^2} \geq 0 \qquad (4)$$

A dedicated Matlab code allows the calculation of the curvature radii, its evaluation relative to 0 and the identification of curve convexity, as function of the supershape defining parameters.

Figure 1 illustrates the block diagram of the algorithm used to determine the variation domains of parameters n_2 and n_3 that lead to convex and convex-concave pitch curves, respectively. In Tab. 2 are presented values of the supershape defining parameters that induce convex-concave pitch curves with 1-3 lobes, scaled to correspond to a gear with 40 number of teeth and modulus m = 2 mm.

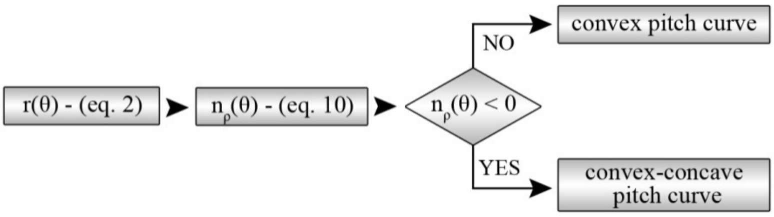


Fig. 1. Determination of the pitch curve’s convexity

Table 2. Variation domains for exponents n_2 and n_3 that lead to convex-concave pitch curves

Semi-axes length	***n***	$n_1 = 1$	$n_1 = 3$	$n_1 = 5$
a* = *b	2	(4.12,15]	(11.8,15]	-
	3	(2.1,15]	(5.44,15]	(8.63,15]
	4	(2.1,15]	(3.06,15]	(5.18,15]
a* > *b	2	(4.12,15]	(11.8,15]	-
	3	(2.1,15]	(5.44,15]	(8.63,15]
	4	(2.1,15]	(3.06,15]	(5.18,15]

A further analysis on the defining parameters variation is developed considering tooth profiles generation with a shaper cutter defined by modulus m and number of teeth z_s. The undercutting appears on the concave region of the curve, characterized by the minimum radius, ρ_{min} (Fig. 2).

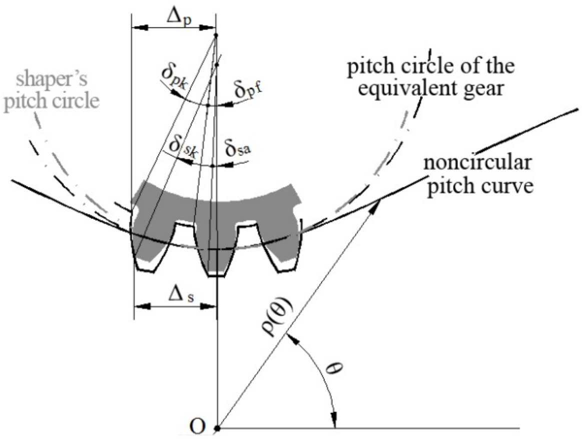


Fig. 2. Undercutting avoidance for teeth generation with a shaper cutter

To avoid undercutting, spur gear generation’s theory imposes that:

$$\Delta_p \geq \Delta_s \tag{5}$$

where

$$\Delta_s = R_{as} \cdot \sin\delta_s \tag{6}$$

R_{as} , the adendum radius of the shaper, is expressed:

$$R_{as} = \frac{z_s}{2} + 1 \tag{7}$$

and

$$\delta_s = \delta_{sk} + \delta_{sa} = = \left(\frac{2\pi}{z_s} \cdot k\right) + \left(\frac{\pi}{2z_s} + inv\alpha - inv\alpha_{as}\right) \tag{8}$$

where k = 1, 2, 3; α_{as} - the pressure angle of the shaper.

$$\Delta_p = R_{fp} \cdot \sin\delta_p \tag{9}$$

R_{fp}, the adendum radius of the equivalent gear, is expressed:

$$R_{fp} = \frac{z_e}{2} - 1 \tag{10}$$

and

$$\delta_p = \delta_{pk} + \delta_{pf} = = \left(\frac{2\pi}{z_e} \cdot k\right) + \left(\frac{\pi}{2z_e} + inv\alpha - inv\alpha_{fp}\right) \tag{11}$$

where k = 1, 2, 3; α_{fp} – pressure angle of the equivalent gear; z_e – teeth number of the equivalent gear:

$$z_e = \left[\frac{2 \cdot \rho_{min}}{m}\right] \tag{12}$$

The validation of Eq. (5) leads to the determination of acceptable variation domains for exponents n_2 and n_3, as shown in Tabs. 3-6. The following concluzions can be drawn:

– as exponent n_1 gets larger values, the acceptable variation domains of exponents n_2 and n_3 increase;

– an increase of the number of lobes implies the need of choosing a larger exponent n_1;

– if parameters *a* and *b* are different and there is an odd number of lobes, open curves are generated.

Table 3. The variation of exponents n_1, n_2, n_3 and pitch curve examples for $a = b$ and $n = 2$

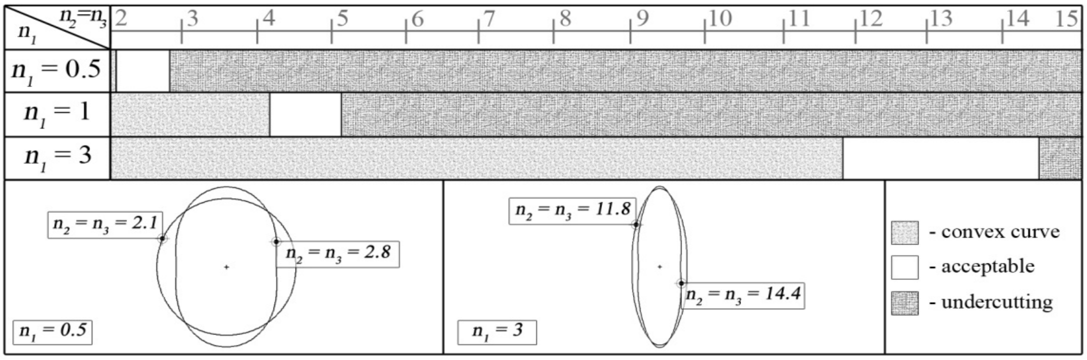


Table 4. The variation of exponents n_1, n_2, n_3 and pitch curve examples for $a = b$ and $n = 3$

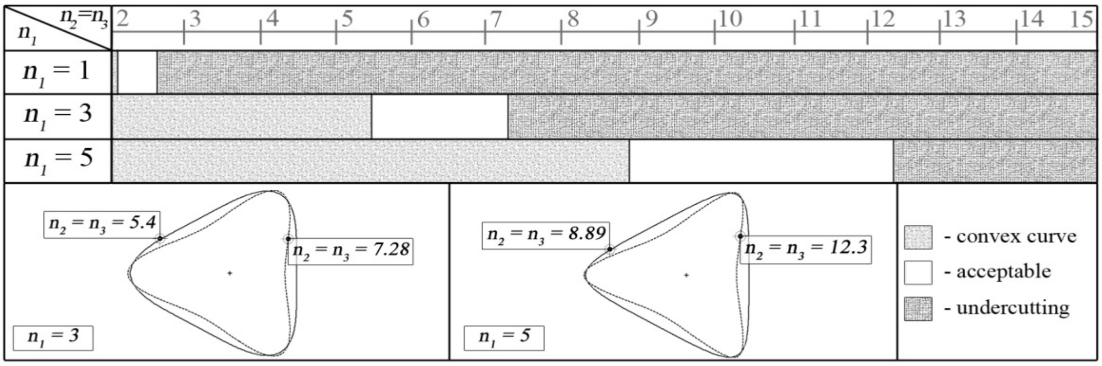


Table 5. The variation of exponents n_1, n_2, n_3 and pitch curve examples for $a \neq b$ and $n = 2$

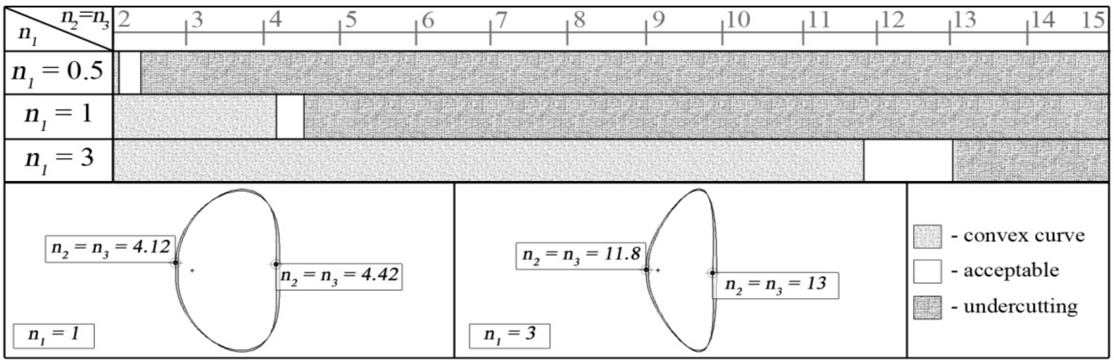
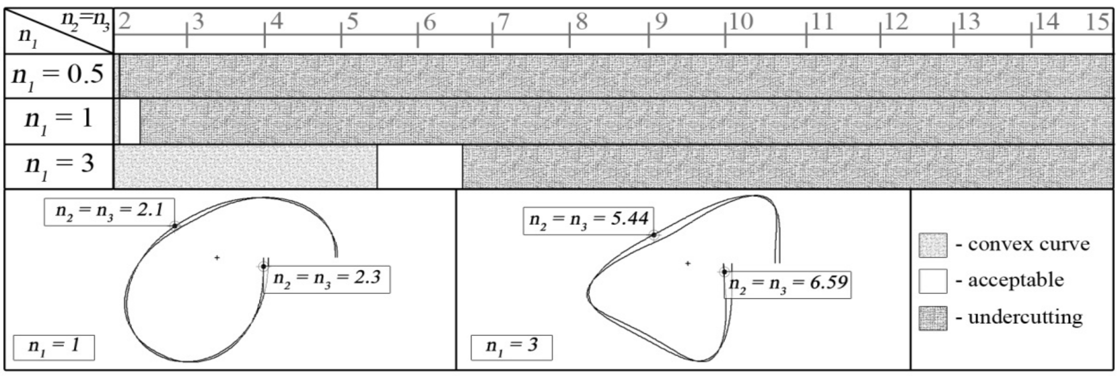


Table 6. The variation of exponents n_1, n_2, n_3 and pitch curve examples for $a \neq b$ and $n = 3$



4. TOOTH GENERATION BY SHAPER

The simulation of gear manufacture process, in case of convex pitch curves, are based on the use of a standard rack cutter and the problem has been covered in another paper [19]. As regard the noncircular convex-concave gear generation, using a shaper cutter, the "cutting process" is related to the following coordinate systems (Fig. 3): $O_fX_fY_f$ is a fixed coordinate system, with the origin in the first point of contact, T_0, between the pitch curves of the gear and tool, respectively, with Y_f axis in the direction of the common tangent to the curves, in T_0; $O_1X_1Y_1$ is a mobile coordinate system, rigidly attached to the gear, with the origin in the polar center of the pitch curve; $O_cX_cY_c$ – a mobile coordinate system, rigidly attached to the shaper, with the origin on X_f axis.

The simulation of the gear generation is based on the following kinematics:

– the gear is rotated around it's center O_1, with angular speed ω_{rr}, respectively angle γ_r, and translated along axes X_f and Y_f, on the distances x_{rt} and y_{rt}, respectively (Fig. 3):

$$\gamma_r(\theta_1) = \theta_1 + \mu(\theta_1) - \pi/2 \tag{13}$$

$$x_{rt}(\theta_1) = r(\theta_1) \cdot \sin\mu(\theta_1) \tag{14}$$

$$y_{rt}(\theta_1) = r(\theta_1) \cdot \cos\mu(\theta_1) \tag{15}$$

where $\mu(\theta_1)$ defines the orientation of the tangent (t) to the gear pitch curve, at current point,

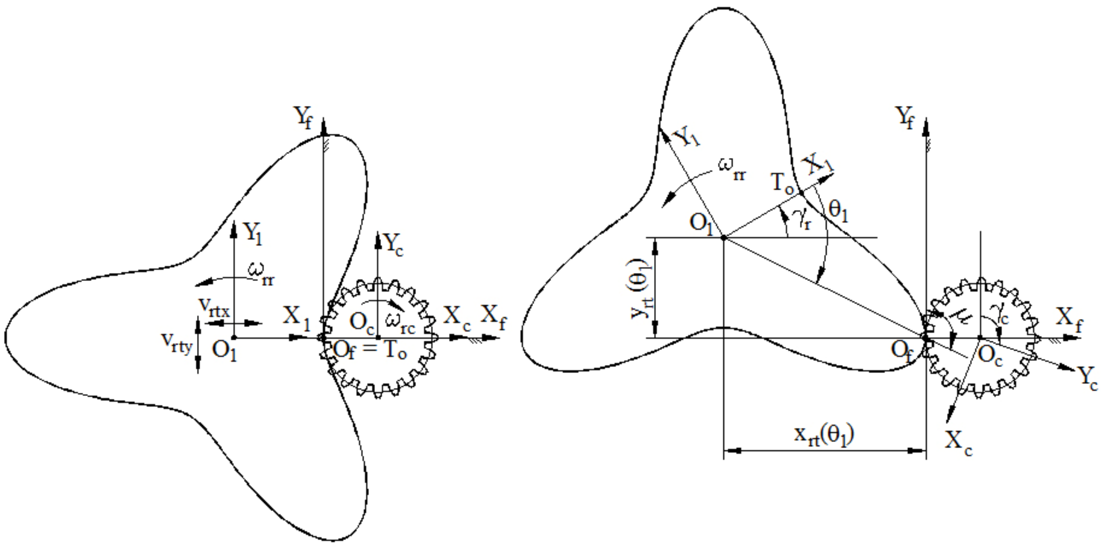
$$\mu(\theta_1) = arctg \frac{r(\theta_1)}{\frac{dr(\theta_1)}{d\theta_1}} \tag{16}$$

– the shaper is rotated with angular speed ω_{cr}, respectively angle γ_c:

$$\gamma_c(\theta_1) = \frac{s(\theta_1)}{R_s} \tag{17}$$

where $s(\theta_1)$ is the distance of rolling, respectively the length of the arc T_0T:

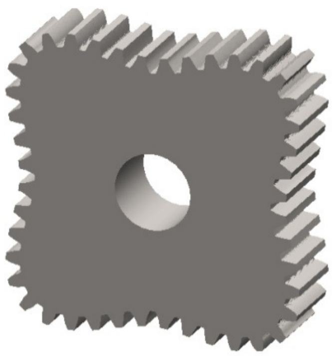
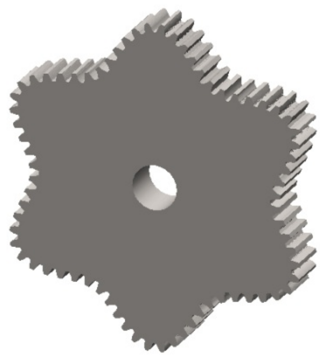
$$s(\theta_1) = T_0T = TO_c = \int_{-\theta_1}^{0} \frac{r(\theta_1)}{\sin\mu(\theta_1)} d\theta_1 \tag{18}$$



a) b)

Fig. 3. Kinematics of gear generation with the shaper in initial position (a) and current position (b)

Table 7. Examples of noncircular gears with convex-concave pitch curves

	
$a = b = 1, n = 4, n_1 = 1, n_2 = n_3 = 3, z = 44$	$a = b = 1, n = 6, n_1 = 1, n_2 = n_3 = 2.8, z = 72$

The kinematics parameters are calculated in Matlab, by a precision of 10^{-15}, and imported in AutoCAD environment were the tooth flanks cutting process is simulated using solid models and boolean operations. The precision of the tooth flank profiles is obviously influenced by the calculus in Matlab, the rotational increment of the shaper and the complexity of the solids manipulated in AutoCAD. Therefore, for certain pitch curve geometries and tooth parameters, the last tooth generated could exhibit an undesire width. Solutions for a proper gear generation simulation consist in a reposition of the shaper in the gear center before any gear movement or in a partial gear generation and multiplication of the gear sector for simmetrical shapes. Examples of noncircular gears generated with a shaper cutter (m = 2 mm, α = 20^o) are presented in Tab. 7.

5. CONCLUSIONS

Three aproaches are mentioned in literature as regard to noncircular gear generation: enveloping theory, analytical generation and simulation of the gear manufacture. In this paper the simulation alternative has been chosen, in the hypothesis of defining the driving pitch curve as Gielis' supershape. Limitation of its six parameters variation is necessary in order to i) exclude pointed shapes and curves with very small curvature radii and ii) avoid undercutting, considering specific tools reccomended by the pitch curve geometry.

The paper is focused on convex-concave gear generation that imposed: identification of gear pitch curve geometry, ie selection of convex and convex-concave curves is automatically made, analysis of defining parameters that allowed proper convex-concave supershapes, that could be used for noncircular pitch curves, and development the simulation of gear generation.

The simulation of gear generation process, by a shaper cutter, is made in AutoCAD environment, based on the interference of Matlab and AutoLISP codes. The complex geometries of the solids manipulated in AutoCAD and numerical approaches for the kinematics parameters calculus in Matlab recommend the simulation of gear generation as a not very precise and fast solution to generate the tooth flanks. For further investigations on gear performances, the authors also developed an analytical method for noncircular gear generation.

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