A GRAPHICAL METHOD DEVELOPED IN THE CATIA DESIGN ENVIRONMENT FOR THE MODELING OF GENERATION BY ENVELOPING

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ABSTRACT

The modelling of surfaces generation is needed for the determination of the geometrical errors, as factor often prevailing in the determination of machining error.

The physical realisation of the tool is accompanied by errors. In this way, the tool’s real shape is different from the theoretical form and, as follows, the generation with this tool’s type is accompanied by an inherent error — the geometrical error. Regarding this, the other factors which lead to the machining error may amplify or, in some cases, may diminish this error.

In this paper, it is proposed a method, developed in the CATIA design environment, for the modelling of the surfaces ordered whirlpool, associated with a couple of rolling centrodes. There are presented numerical examples for applying of the proposed algorithm.

KEYWORDS: CATIA, surfaces ordered whirlpool, surfaces generation

1. Introduction

They are known the fundamentals theorems of the surfaces generation [1], [2], from which is based the determination of the generating tool’s profiles. These theorems are the fundamentals of the generation modelling, when, knowing the tool’s form, it is possible to determine the real generated surface’s form [5], [6], [7].

The description of the tool’s primary peripheral surface is done in numerical form, because this surface may be known by direct measuring on 3D coordinate measuring machines. The surfaces are obtained as point clouds. A simplified solution is this when it is known a surface’s generatrix, usually an in-plane curve, representing an axial (or crossing) section of the tool’s primary peripheral surface. With this it is generated a virtual primary peripheral surface of the tool, which is used for the subsequent modelling of the generated piece.

The specific form in which it is known the tool’s primary peripheral surface (points cloud) require the uses of a specific enveloping method, in the virtual generating process, for the determination of the tool’s surface enveloping — the method of the minimum distance [4].

Obviously, the proposed solution for the determination of the generating geometrical error has some limits, but can constitute an alternative for the virtual control, before the effective machining of the generated surface.

Also, the proposed solution may be used for the determination of the constructive modifications of the tool, in order to diminish the geometrical errors and, implicitly, the machined surface’s generating error.

2. The modelling of the surfaces generation at machining with rack-gear tool

The issue of the generation with the rack-gear tool has to be regarded as an “inverse” problem regarding the tool’s profiling.

It is known, in numerical form, the rack-gear tool’s profile and, in the centrodes rolling motion, it is determined the profile’s form of one of the surfaces to be generated — the virtual model of the surface.

The generating kinematics is known, see Fig. 1.

The following parameters are defined:

C_1 is the piece’s centrode, a circle arc with R_{rp} radius, associated with the whirlpool to be generated;

C_2 — straight line centrode, associated with the S profile of the rack-gear tool;

$\xi\eta\zeta$ — relative reference system, joined with the rack-gear tool;
XYZ — relative reference system, joined with the blank;
xyz — global reference system.

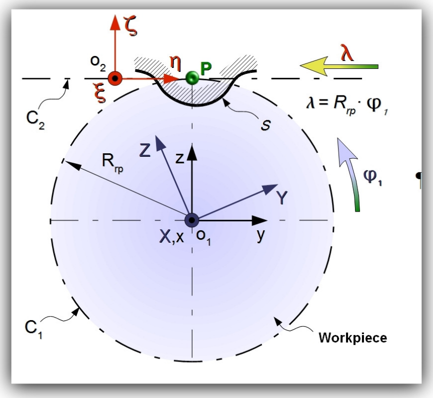

Fig. 1. *The rack-gear tool's profile and the rolling centrodes*

2.1. The generating process kinematics

There are defined the rack-gear absolute motions and the space associated with the blank, regarding the global reference system:

$$x = \omega_1^T(\varphi_1) \cdot X \tag{1}$$

and

$$x = \xi + \begin{pmatrix} 0 \\ -R_{rp} \cdot \varphi_1 \\ R_{rp} \end{pmatrix} \tag{2}$$

where φ_1 is the angular motion parameter around the *x* axis.

Now, based on the (1) and (2) movements, it is possible to determine the relative motion:

$$X = \omega_1(\varphi_1) \cdot \left[\xi + \begin{pmatrix} 0 \\ -R_{rp} \cdot \varphi_1 \\ R_{rp} \end{pmatrix} \right] \tag{3}$$

of the rack-gear tool regarding the blank's reference system.

If it is assumed as known the rack-gear tool's profile, in numerical form, a matrix of points

$$S = \begin{pmatrix} 0 \\ \eta_i \\ \zeta_i \end{pmatrix}_{i=1...n} \tag{4}$$

then, from (3), it is possible to determine an array with coordinates of points belonging to the profile's family *S*

$$\begin{pmatrix} X_{i,k} \\ Y_{i,k} \\ Z_{i,k} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_1 & \sin\varphi_1 \\ 0 & -\sin\varphi_1 & \cos\varphi_1 \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ \eta_i \\ \zeta_i \end{pmatrix} + \begin{pmatrix} 0 \\ -R_{rp} \cdot \varphi_1 \\ R_{rp} \end{pmatrix} \right]_{(i=1...n);(k=1...m)} \tag{5}$$

for an incremental variation of the φ_1 parameter,

$$\varphi_1 = k \cdot \Delta\varphi_1 \tag{6}$$

where $k=1,2,...n$ and $\Delta\varphi_1$ is the increment.

The Σ profile of the surface is determined by associating with the point's coordinates array obtained from (5),

$$S_{i,k} = \begin{pmatrix} 0 \\ Y_{i,k} \\ Z_{i,k} \end{pmatrix}_{(i=1...n);(k=1...m)} \tag{7}$$

a form of the enveloping condition, the method of "the minimum distance" [4], see Figure 2. The enveloping condition is:

$$d_{min} = \left\{ \left[\left[Y_{(i,k)} - R_{rp} \sin(k \cdot \Delta\varphi_1) \right]^2 + \left[Z_{(i,k)} - R_{rp} \sin(k \cdot \Delta\varphi_1) \right]^2 \right]^{1/2} \right\}_{min} \tag{8}$$

where:

$$i = 1, 2, ...n; \; k = 1, 2, ...m; \quad P \left| \begin{array}{l} Y_P = R_{rp} \sin(k \cdot \Delta\varphi_1); \\ Z_P = R_{rp} \cos(k \cdot \Delta\varphi_1), \end{array} \right. \tag{9}$$

Y_P and Z_P are the coordinates of the gearing pole, *P*, see Figure 2.

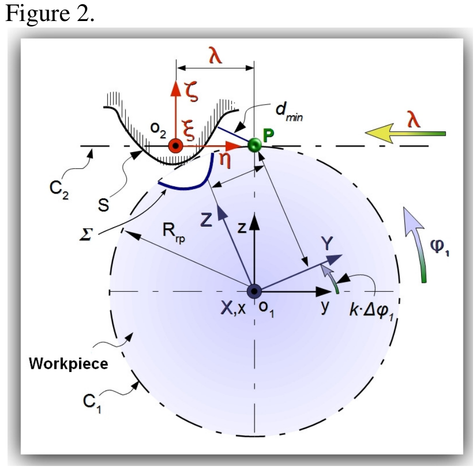

Fig. 2. *Inverse generation (the method of "minimum distance")*

All the points belonging to the array (7), for which, in various rolling positions, the distance to the gearing pole, *P*, is minimum, represent the virtual profile, effectively generated, of the blank — the *Σ* effective profile.

In the following subchapter it is proposed a kinematical method for the determination of the surface effectively generated by rack-gear tool.

3. The graphical method for the modelling of a polygonal shaft

In the CATIA design environment, the problem consist in the determination of the crossing section of the 3D model resulted by running an application, *MGMC-VBA*, which simulates the machining process for a certain increment of the blank rolling motion.

As example it is considered the polygonal shaft presented in Figure 3, representing the theoretical solid model of the shaft to be generated.

For this shaft, was determined the crossing section of the rack-gear, by running a dedicated application. In this application were given the input parameters representing the shaft external diameter, the number of sides (*NbS*=6) and $R_e=R_r$=50 mm.

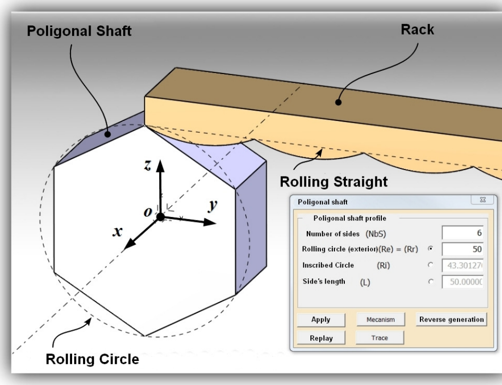

Fig. 3. *The theoretical polygonal shaft and the theoretical rack-gear resulted from the MGMC-VBA application*

In the Table 1, are presented the coordinates of points from the theoretical profile of the rack-gear.

Table 1. *Coordinates of points from the rack-gear tool's profile*

		ξ[mm]	*η[mm]*			*ξ[mm]*	*η[mm]*
Erroneous profile	***1***	0,000	0,000	***Theoretically profile***	***1***	0,000	0,000
	2	5,484	-2,573		***2***	5,446	-2,676
	3	11,216	-4,534		***3***	11,183	-4,656
	4	17,129	-5,852		***4***	17,110	-5,966
	5	23,151	-6,516		***5***	23,145	-6,617
	6	29,209	-6,516		***6***	29,215	-6,617
	7	35,231	-5,852		***7***	35,250	-5,966
	8	41,143	-4,534		***8***	41,176	-4,656
	9	46,876	-2,573		***9***	46,913	-2,676
	10	52,360	0,000		***10***	52,360	0,000

In Figure 4, it is presented the 2D model of the rack-gear tool, modified regarding its theoretical model (Table 1), due to the errors resulting from the machining process of from the using of these.

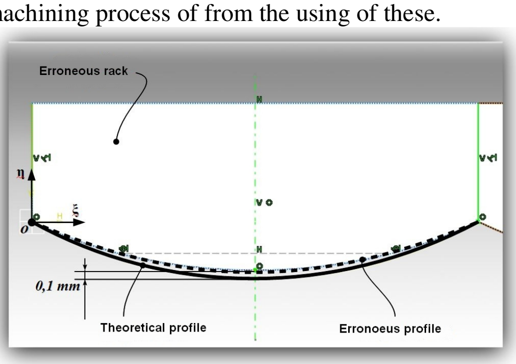

Fig. 4. *Rack-gear with errors — 2D model*

It is desired to determine the profile of the crossing section of the 3D model of the virtual shaft which will result from the simulation of the machining process with this erroneous rack-gear tool.

For this, it is considered the solid model of the blank, represented by a cylinder with radius bigger than the peripheral radius of the polygonal shaft to be generated. It is run the *MGMC-VBA* application, and are given the rolling angles and the rotation angle increment for each step of the virtual machining process simulation (see Figure 5).

The specific *MGMC-VBA* application automates the copy and pastes commands of the solids in the corresponding positions, rotated and respectively translated (*Copy*, *Paste as result*, *Rotate* and *Translate*) as well as the generating process by extracting of the tool’s solid (command *Remove*), from the virtual blank.

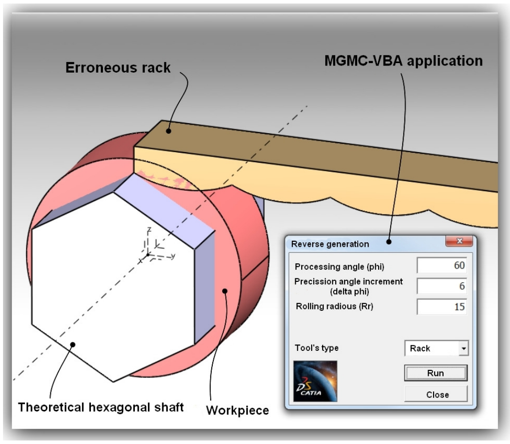

Fig. 5. *Inverse generation. The MGMC-VBA application*

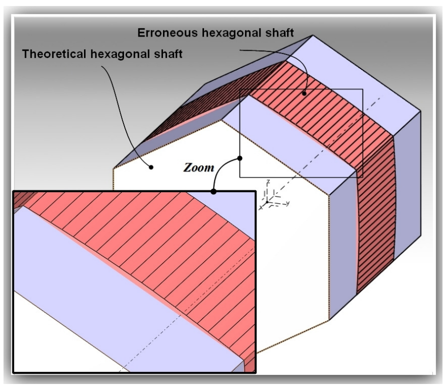

Fig. 6. *Solid model of the theoretical and erroneous hexagonal shaft*

As far as the virtual generation is concerned, it is analysed the crossing section profile of the shaft resulted in the *YZ* plane, see Figures 6 and 7. This profile is obtained with the *Intersection* command, obtaining the coordinates of points from this profile.

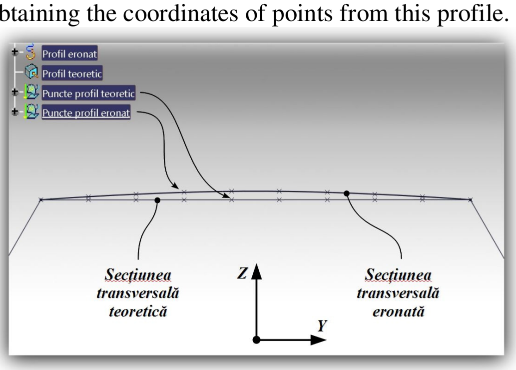

Fig. 7. *Theoretical and erroneous crossing sections of the shaft*

In Table 2, we present the coordinates of profile generated with the real (erroneous) rack gear, regarding the target profile (the theoretical profile) of the hexagonal shaft.

Table 2. *Erroneous profile versus theoretical profile of the hexagonal shaft*

	Y[mm]	*Z[mm]*		*Y[mm]*	*Z[mm]*
Theoretical profile	43.301	25.000	***Erroneous profile***	43.301	25.000
	43.408	19.445		43.301	19.444
	43.428	13.890		43.301	13.889
	43.417	8.334		43.301	8.333
	43.403	2.778		43.301	2.778
	43.403	-2.778		43.301	-2.778
	43.417	-8.334		43.301	-8.333
	43.428	-13.890		43.301	-13.889
	43.408	-19.446		43.301	-19.444
	43.301	-25.000		43.301	-25.000

The coordinates of points from this profile may be exported and analysed.

4. The graphical method for the modelling of an involute teethed wheel

It is presented the example of modelling for the generation of a cylindrical teethed wheel with straight teeth made with the *MGMC-VBA* application.

In Figure 8, it is presented the solid model of the teethed wheel for the parameters from the Table 3.

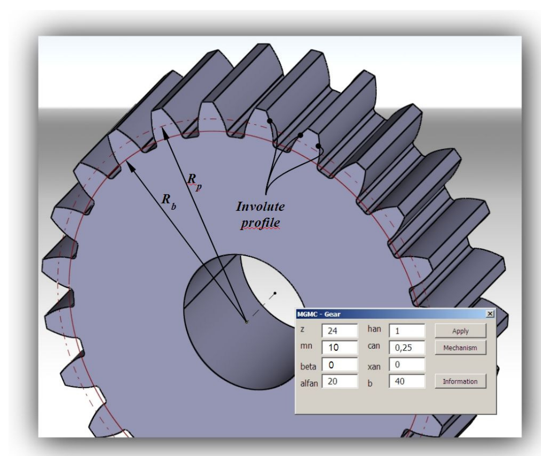

Fig. 8. *The theoretical teethed wheel and the MGMC-VBA application*

Table 3. *Erroneous profile versus theoretical profile of the hexagonal shaft*

Parameter	***Symbol***	***Value***
Number of teeth	z	24
Normal module	mn	10 mm
Tooth angle	beta	0 °
Working normal pressure angle	alfan	20 °
Addendum coefficient	han	1
Dedendum coefficient	can	0,25
Rack shifting coefficient	xan	0
Gear facewidth	b	40

On the Starrett Optical horizontal profile projector were measured 200 points from the wear rack-gear which was used for the machining of a teethed wheel similar to this one modelled in the CATIA design environment.

The coordinates of these points, see Table 4, were imported in CATIA using the *MGMC-VBA* application, obtaining the real profile of the measured rack-gear.

Table 4. *Coordinates of points from the measured rack-gear*

Crt. no.	***X [mm]***	***Y [mm]***	***Crt. no.***	***X [mm]***	***Y [mm]***	***Crt. no.***	***X [mm]***	***Y [mm]***
1	0,041	0,078	***80***	35,508	-1,009	***160***	74,401	-20,949
2	0,325	0,004	***81***	36,061	-2,479	***161***	74,89	-22,271
3	0,829	0,010	***82***	36,61	-3,981	***162***	75,428	-23,722
4	1,326	0,008	***83***	37,074	-5,228	***163***	75,919	-25,034
5	1,825	0,039	***84***	37,622	-6,717	***164***	76,149	-25,519
6	2,325	0,035	***85***	38,087	-7,984	***165***	76,309	-25,645
7	2,622	0,032	***86***	38,68	-9,589	***166***	76,419	-25,68
8	2,901	-0,011	***87***	39,219	-11,036	***167***	76,927	-25,698
9	3,200	-0,086	***88***	39,811	-12,602	***168***	77,493	-25,691
...	...	...	***...***	...	...	***...***	...	...

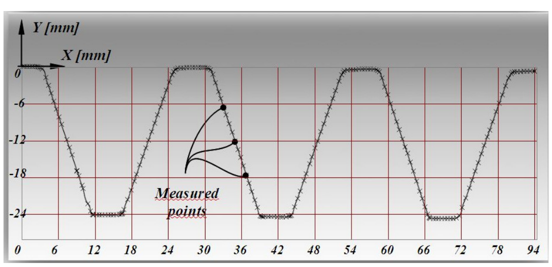

Fig. 9. *Measured points from the rack-gear profile*

The goal is to model the teethed wheel generated with the rack-gear with the real profile presented in Figure 10. In order to make this we modelled the following solids: the theoretical teethed wheel; the real rack-gear and a cylindrical blank, coaxially with the theoretical teethed wheel, see Figure 10.

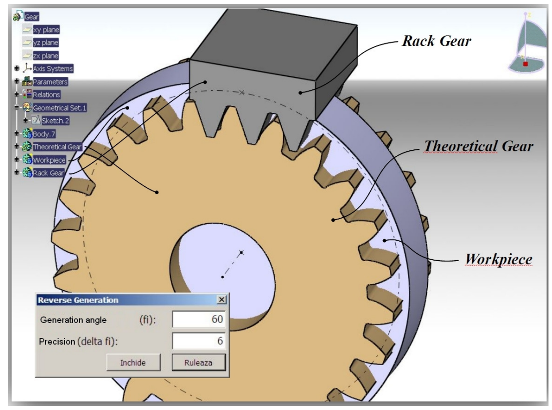

Fig. 10. *Solid models of the teethed wheel, rack-gear and cylindrical blank*

Running the *MGMC-VBA* application (*Inverse generation*), with the application’s parameters: generating angle (*fi=60°*) and the increment of the blank rotation angle (*deltafi=6°*), it was obtained the solid model of the teethed wheel virtually machined with the measured rack-gear.

In order to compare the two solid models of the teethed wheels, the theoretical one with the modelled one, were considered the intersections between these and three concentric circles with radius: R_1=115 mm; R_2=120 mm and R_3=125 mm, in a crossing plane.

The intersection points between these circles and the teethed wheels’ profiles are presented in Figure 11.

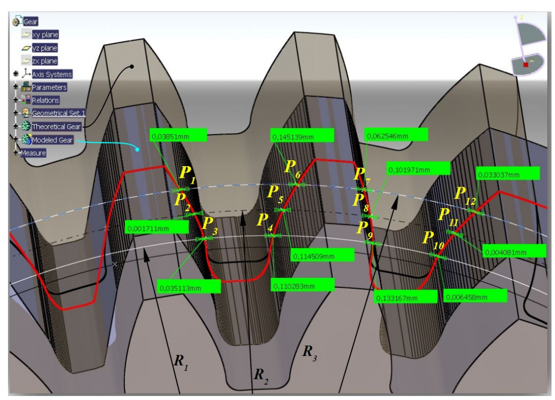

Fig. 11. *Virtually machined teethed wheel and the theoretical teethed wheel—Errors*

The minimum distances between the couples of points corresponding for the two profiles, representing the generating errors of the modelled teethed wheel regarding the theoretical one, are presented in Table 5.

Table 5. *Errors between the two teethed wheel, the theoretical one and the modeled one*

Crt. no.	Modeling errors [mm]
P_1 P_1’	0,039
P_2 P_2’	0,002
P_3 P_3’	0,035
P_4 P_4’	0,110
P_5 P_5’	0,115
P_6 P_6’	0,145
P_7 P_7’	0,063
P_8 P_8’	0,102
P_9 P_9’	0,133
P_{10} P_{10}’	0,006
P_{11} P_{11}’	0,004
P_{12} P_{12}’	0,033

5. Conclusions

The 3D solid modelling method of generation with the rack-gear tool, in the CATIA design environment, was done using an in-house application made for this problem.

It was presented an application example which proves the capability of this method to rigorously solve the problem of 3D modelling of generation with rack-gear tool.

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Metodă cinematică în mediul grafic CATIA pentru modelarea generării prin rulare

—Rezumat—

Problema modelării generării suprafeţelor este importantă în vederea determinării erorilor geometrice de generare, ca factor, adeseori, dominant în determinarea erorii de prelucrare.

Realizarea fizică a sculei este însoțită de erori. În acest fel, forma reală a sculei este diferită de forma teoretică a acesteia şi, ca urmare, generarea cu o astfel de sculă este însoțită de o eroare inerentă — eroare geometrică. În raport cu aceasta, ceilalți factori care conduc la apariția erorii de prelucrare o pot amplifica sau, în situații speciale, o pot diminua.

În lucrare, se propune un model, dezvoltat în mediul grafic CATIA, de modelare a vârtejurilor ordonate de suprafețe (profiluri) asociate unui cuplu de centroide în rulare. Sunt prezentate exemple numerice de aplicare a algoritmului propus.