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AN ALGORITHM FOR REGISTRATION OF POINTS MEASURED ON COORDINATE MEASURING MACHINE

Virgil Teodor, Ionuț Popa, Șchiopu Doru

"Dunărea de Jos" University of Galați
virgil.teodor@ugal.ro

ABSTRACT

In the measuring process of a part on coordinate measuring machines (CMM), one of the frequent problems is the necessity to modify the part position during measuring operation, so the measuring system has access to the various compounding surfaces. This thing assumes the modification of the piece reference system initially established and requires the reestablishment of the reference system or the establishing of a new reference system. Sometimes, due to the new position of the part, it is impossible to achieve the features used for the creation of the initial reference system. Moreover, the simultaneous measuring of the two necessary reference systems may be impossible.

In this paper, are proposed an algorithm and a software which allow for the correlation of the points measured in two distinct stages, respectively in the two different positions of the same part. The algorithm is based on the determination of three control points in the two distinct measuring positions and then the establishing of the coordinate transformation which determines the modified position of the measured part.

KEYWORDS: coordinate measuring machines, registration, genetic algorithm

1. Introduction

Now, the coordinates measuring machines (CMM) allow for the measuring of coordinates of a point from the part's surfaces, the establishing of a piece reference system and the mathematical correlation between this reference system and the machine reference system (usually called world reference system) [3], [4], [5].

The correlation process between the two reference systems (the piece system and the world system) is called alignment [8].

In metrology, the measuring result is a feature of a part, for example a bore, a surface or a gap. The measuring of a part is used to determine the distance and position of a feature on this type, regarding another element [1], [2].

One of the frequent problems emerged in the measuring process is the necessity to modify the position of the part during measurement, so the touch probe has access to the various features of the part. Another possibility is to change the position of the touch probe in the machine measurement system.

By any of these modifications the initially established reference system is lost and emerges the necessity to re-establish this reference system or to establish a new reference system used for referring to the new features which will be measured [6].

Probably, due to the new position of the part, the access of the touch probe to the features based on which was established the first reference system is not possible. Also, probably the features needed for the establishing of the two reference systems is impossible to be simultaneously measured.

An example in this sense is the case of a part having as functional surfaces two holes with disjoint axis (surfaces S_1 and S_2), see Figure 1.

In the presented example, the external surfaces of the part do not have a functional role and so they do not need a precise machining from the dimensional and surfaces quality points of view. Obviously the establishing of a reference system based on the part's external surfaces will be very imprecise and would not be justified from the metrological point of view.

A solution for this problem is the inspection of the two surfaces in convenient positions for each of them and then the refer of the features position at a unique reference system.

The correlation process of the two features is called registration.

Furthermore, it is proposed a method which allows for the correlation of points from the surfaces measured in two distinct stages, respectively in two different positions of the same part. The method is based on the determination of the coordinates of three control points and then the establishing of the coordinate transformation upon which was obtained the modified position of the part to be measured.

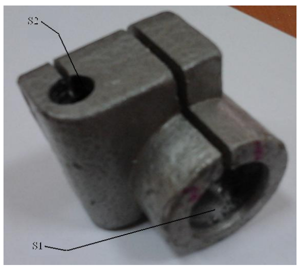

Fig. 1. *Part with two functional surfaces*

2. Principle of the method

It is considered a point *P* on one of the part's surfaces, with coordinates (*x,y,z*) in the world reference system. The *P'* position of the same point will be characterised by coordinates (*X,Y,Z*) in the piece's reference system. Although *P* and *P'* represent physically the same point from the measured surface, the reference to the different reference systems leads to distinct coordinates sets.

The registering is the process of determination of the coordinate transformation which links the two reference systems. The equation of this coordinate transformation is

$$P = P' \cdot R + T, \qquad (1)$$

where the *R* matrix is the rotation matrix and *T* is the translation vector.

The problem consists in fact that the *R* and *T* are unknown. The registering algorithm allows for the determination of these values based on the overlapping of the three points' position.

The development of the equation (1) leads to a matrix equation such as type:

$$x = \omega_3(\varphi) \cdot \omega_2(\psi) \cdot \omega_1(\chi) \cdot X + x_0, \qquad (2)$$

or, developed

$$\begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{Vmatrix} \cdot$$
$$\cdot \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & \sin\chi \\ 0 & -\sin\chi & \cos\chi \end{Vmatrix} \cdot \begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} + \begin{Vmatrix} x_0 \\ y_0 \\ z_0 \end{Vmatrix}, \qquad (3)$$

where: φ is the rotation angle around the *z* axis;
ψ — rotation angle around the *y* axis;
χ — rotation angle around the *x* axis;
x_0, y_0, z_0 — coordinates of the *X,Y,Z* reference system's origin in the *x,y,z,* world reference system.

This coordinate transformation is presented in Figure 2.

The equation of the inverse transformation (from the *x, y, z* reference system to the *X,Y,Z* reference system is given by:

$$X = \omega_1^T(\chi) \cdot \omega_2^T(\psi) \cdot \omega_3^T(\varphi) \cdot (x - x_0), \qquad (4)$$

or developed:

$$\begin{Vmatrix} Y \\ Y \\ Z \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & -\sin\chi \\ 0 & \sin\chi & \cos\chi \end{Vmatrix} \cdot \begin{Vmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{Vmatrix} \cdot$$
$$\cdot \begin{Vmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \left(\begin{Vmatrix} x \\ y \\ z \end{Vmatrix} - \begin{Vmatrix} x_0 \\ y_0 \\ z_0 \end{Vmatrix} \right). \qquad (5)$$

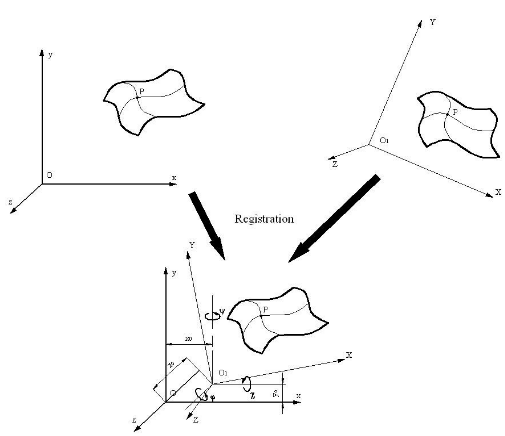

Fig. 2. *Coordinate transformation*

Obviously, in equation (3) are involved six unknown values (φ,ψ, χ, rotation angles around the x0, y0, z0, axis of the world reference system and x0, y0, z0, distances between the origins of the two reference systems, measured along the axis of the world reference system).

For the determination of these unknown values, it is needed an equation system with six equations.

This equations system is obtained by measuring the coordinates of the three points in the two reference systems previously mentioned.

For the three points, the equation (3) leads to the system with six equations and six unknown values:

$$\begin{cases} x_1 = X_1\cos\varphi\cos\psi + Y_1(\sin\varphi\cos\chi+\cos\varphi\sin\psi\sin\chi) + Z_1(\sin\varphi\sin\chi-\cos\varphi\sin\psi\cos\chi) + x_0; \\ y_1 = -X_1\sin\varphi\cos\psi + Y_1(\cos\varphi\cos\chi-\sin\varphi\sin\psi\sin\chi) + Z_1(\cos\varphi\sin\chi+\sin\varphi\sin\psi\cos\chi) + y_0; \\ z_1 = X_1\sin\psi - Y_1\cos\psi\sin\chi + Z_1\cos\psi\cos\chi + z_0; \\ x_2 = X_2\cos\varphi\cos\psi + Y_2(\sin\varphi\cos\chi+\cos\varphi\sin\psi\sin\chi) + Z_2(\sin\varphi\sin\chi-\cos\varphi\sin\psi\cos\chi) + x_0; \\ y_2 = -X_2\sin\varphi\cos\psi + Y_2(\cos\varphi\cos\chi-\sin\varphi\sin\psi\sin\chi) + Z_2(\cos\varphi\sin\chi+\sin\varphi\sin\psi\cos\chi) + y_0; \\ z_2 = X_2\sin\psi - Y_2\cos\psi\sin\chi + Z_2\cos\psi\cos\chi + z_0; \\ x_3 = X_3\cos\varphi\cos\psi + Y_3(\sin\varphi\cos\chi+\cos\varphi\sin\psi\sin\chi) + Z_3(\sin\varphi\sin\chi-\cos\varphi\sin\psi\cos\chi) + x_0; \\ y_3 = -X_3\sin\varphi\cos\psi + Y_3(\cos\varphi\cos\chi-\sin\varphi\sin\psi\sin\chi) + Z_3(\cos\varphi\sin\chi+\sin\varphi\sin\psi\cos\chi) + y_0; \\ z_3 = X_3\sin\psi - Y_3\cos\psi\sin\chi + Z_3\cos\psi\cos\chi + z_0. \end{cases} \quad (6)$$

In the equations (6), x_i, y_i, z_i ($i=1\div3$) represent the coordinates of the point P_i in the world reference system and X_i, Y_i, Z_i the coordinates of the same point in the piece's reference system.

By solving the equations system (6), are determined the values of the φ, ψ, χ and x_0, y_0, z_0, parameters. This fact allows knowing the coordinates of a certain point P in one of the reference system when its coordinates are known in the other reference system.

In practice, after the measurement of the surfaces which can be inspected with the touch probe in a certain position of the part, will be measured three control points positioned so that these points can be achieved in the both measuring positions of the part.

The part is repositioned and are measured these surfaces which can not be achieved in the initial measuring position, then, without moving the part, they are measured again the three control points.

By solving the equation system (6) for the coordinates of the three control points obtained in two distinct positions of the part, it is possible to determine the coordinate transformation which links the reference system associated with the two measurements. Now it is possible for the expression of the part surfaces to be made in a unique reference system and to be determined the relative position of the functional surfaces.

Note: It is important for the three points not to be collinear.

Also, we have to bear in mind that the measurement of the three points in the same order eases the registration process.

In case that the two measuring positions not allow to select three points which can be measured in the both positions it is possible to use an auxiliary device to materialize the control points and which will be mounted onto the part through some method, in a position accessible for the both measurement stages (see Figure 3).

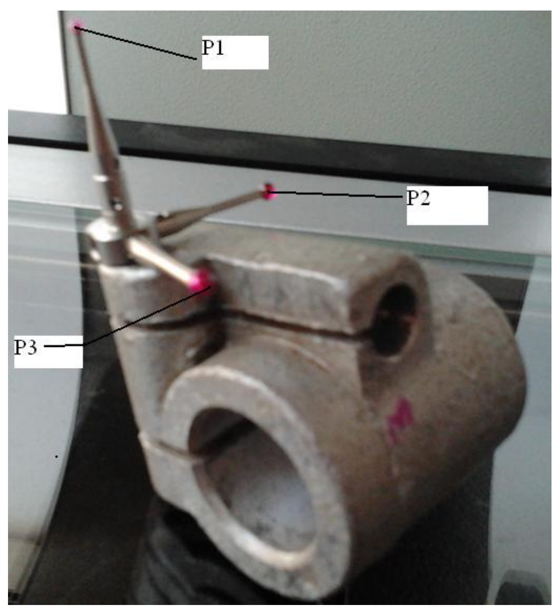

Fig. 3. *Materialization of the P_1, P_2 and P_3 control points using an auxiliary device*

3. The problem solving

Solving the problem by usual mathematical methods is difficult due to the complicated form of the resulted equation (see equations (6)).

The solving of this equations system with six unknown values is possible using one of the following methods: the genetic algorithm or the method of parameter circulation [5], [6], [7].

3.1. The genetic algorithm method

In this case, it is considered as objective function the sum of the squared distances calculated between two sets of points. The first points set is obtained applying to the coordinates of points obtained by initial measurement a reference system transformation given by equations (3). The second point set is obtained by the second measurement. Obviously the sum of squared distances will be minimal when the parameters of transformation are equals with the real parameters.

Admitting that at the first measurement are determined the coordinates of points P_1, P_2 and P_3, by the coordinate transformation (3) are obtained the coordinates of the same points in a new reference system. We denote with X_i, Y_i and Z_i ($i=1\div3$) these coordinates in the first reference system.

If we note with x_i, y_i and z_i, ($i=1\div3$), the coordinates of p_1, p_2 and p_3 points in the new reference system, the objective function will be given by equation:

$$d=\sum_{i=1}^{3}\sqrt{\left(x_i-X_i\right)^2+\left(y_i-Y_i\right)^2+\left(z_i-Z_i\right)^2}\ . \quad (7)$$

By optimising this objective function so that it is obtained a value as close to *0* as possible are determined the transformation parameters from one reference system to other.

This way to solve the problem allows finding a solution for the equations system (6), but the limitation linked to the application of this algorithm refers to those characteristic for the genetic algorithm.

As it is known, the genetic algorithms are heuristics algorithms, so the solution found is not always the optimal solution but it is in a close to of the optimal solution.

The time needed for the run of the genetic algorithm depends on the allowed level for error. In case of precision requirements usually from the technical point of view the running time may be relatively high.

At two distinct runs of the algorithms it is possible to obtain a distinct solution (but closed in the error limit).

3.2. The parameters circulation method

The method of parameters circulation consists in the exhaustive searching of the model’s parameters values, in a small space, around a set of values initially established in an arbitrary way. This set of values determines the optimal similitude between the model and the real element.

For the analysed case, the mathematical model is the coordinate transformation and the parameters of this model are φ, ψ, χ, rotation angles around the world reference system’s axis and the x_0, y_0, z_0, distances between the origins of the two reference systems along the world reference system’s axis.

In this case, the objective function will be given by equation (7), where the points coordinates x_i, y_i and z_i, ($i=1\div3$), are calculated with coordinate transformation (2).

The partial derivatives of the objective function are calculated regarding each of the model’s parameters.

By slight modifications of the value of each parameter, it is obtained a value set for which each of the derivative has a minimal value.

When this set of values corresponds to an acceptable value of the objective function, we can consider that the optimisation process is concluded and this set of values represents the parameters of the coordinate transformation (the model’s parameters).

Although the precision given by this method is high enough and the algorithm is highly convergent, the method has the disadvantage that the running time depends largely on the initial set of values.

4. Numerical application

In order to verify the proposed algorithm it was simulated the measurement of 3 points with coordinates: P_1 *[0;0;0]*, P_2 *[100;0;0]*, P_3 *[0;100;0]*.

For these points was applied the coordinate transformation with parameters: φ=0.1 rad, ψ=0.2 rad, χ=0.3 rad, x0=10 mm, y0=20 mm şi z0=30 mm.

In this way were obtained the transformed points’ coordinates, having the values: p_1 *[10; 20; 30]*, p_2 *[103.63; -7.5096; 51.835]*, p_3 *[38.963; 115.64; 26.304]*.

Based on the initial coordinates of points and the final points coordinates were applied the two presented methods.

In both cases it was considered as admissible the value of the objective function $\varepsilon=0.01$ mm.

In the case of genetic algorithm method was used the *Genetic Algorithm Tool* software in the frame of the *Matlab* software, version 7, with options *Population Size*=20; *Elite Count*=2; *Crossover Fraction*=0.8; *Migration Interval*=20; *Migration Fraction*=0.2.

The running of program was made for 200000 generations and the value of the objective function at the running end was $\varepsilon=0.009$ mm. The running time was about 75 minutes.

In the case of parameter circulation was used a, in-house software developed also in *Matlab* version 7.

The program running assumes 23 cycles for parameters modification. The value of the objective function was $\varepsilon=0.0087$ mm. The running time was about 5 minutes.

In Table 1, are presented the relative errors obtained for the previously presented cases.

Table 1. *Relative errors for the method of genetic algorithm and for the method of parameters circulation*

Parameter	Theoretical value	Value obtained by genetic algorithm	Value obtained by parameters circulation	Relative error for GA	Relative error for PC
φ	0.1	0.0997	0.1	0.3	0
ψ	0.2	0.2006	0.1999	0.3	0.05
χ	0.3	0.2996	0.3	0.133333	0
x_0	10	10.0184	9.9989	0.184	0.011
y_0	20	19.982	20.0022	0.09	0.011
z_0	30	29.9747	30.0035	0.084333	0.011667

It is obvious that in both cases the relative error of parameters is small enough to be used in practice.

5. Conclusions

In this paper is proposed an algorithm which allows correlating points from two surfaces measured in distinctive measurement stages. The algorithm is based on the determination of three control points in the two measurement positions and the establishing of the coordinate transformation which determines the part’s modified position.

Based on this algorithm, were created two methods for the determination of the coordinate transformation and were created an in-house software which allows for the solving of this problem.

The numerical results prove that the proposed methods are precise enough to be used in industrial practice.

It was observed that the method based on the parameters circulation is more exact and faster than the method based on the genetic algorithms.

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Algoritm de corelare a punctelor măsurate pe maşini de măsurat în coordonate

—Rezumat—

În procesul de măsurare a unui reper pe maşinile de măsurat în coordonate (CMM), una dintre problemele frecvent apărute este necesitatea de a modifica poziția reperului în timpul măsurării, astfel încât sistemul de măsurare să aibă acces la diversele suprafețe componente. Acest lucru presupune modificarea sistemului de referință al piesei stabilit inițial şi necesită fie restabilirea acestui sistem de referință, fie stabilirea unui nou sistem de referință. Uneori, datorită noii poziții a reperului, este imposibil accesul la acele elemente pe baza cărora a fost creat sistemul de referință inițial. Mai mult poate apare imposibilitatea de a măsura simultan cele două sisteme de referință necesare (cel inițial şi un nou sistem de referință la care să fie raportate suprafețele ce urmează a fi măsurate).

În această lucrare este propusă o metodă şi programe de calcul care să permită corelarea punctelor de pe suprafețe măsurate în două etape distincte, respectiv în două poziții diferite ale aceluiaşi reper. Metoda este bazată pe determinarea coordonatelor a trei puncte de control în cele două poziții distincte de măsurare şi apoi stabilirea transformării de coordonate pe baza căreia s-a ajuns la poziția modificată a reperului de măsurat.