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FINITE-TIME THERMODYNAMICS BASED ECOLOGICAL ANALYSIS OF AN IRREVERSIBLE RANKINE HEAT ENGINE

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ABSTRACT

Ecological analysis using finite-time thermodynamics of an irreversible Rankine heat engines is reported in this paper. The effects of the temperatures differences between the engine and thermal reservoirs, sizes of heat exchangers, heat capacity rates of heating and cooling fluid and degree of internal irreversibility on the maximum ecological function are discussed.

KEYWORDS: finite-time thermodynamics, heat engines, ecological analysis

1. Introduction

In 1977, Anderson et al. introduced the name *finite time thermodynamics* (FTT) [1]. Finite-time thermodynamics uses more realistic models than those provided by classical thermodynamics and offers a deeper understanding of how irreversibility affects the performance of heat engines. The evolution of finite-time thermodynamics was imposed by the necessity to produce power and to consider heat transfer interaction with the surrounding. The power produced by a heat engine is work divided by time. Considering reversible processes (which are carried out at an infinitely slow pace) the power generated by the engine is zero because a finite amount of work produced by the engine over an infinite time does not deliver power. The origin of finite-time thermodynamics are in three works prepared independently, one by Novikov, one by Chambadal and one by Curzon and Ahlborn. These authors considered that the heat transfer to and from a Carnot engine has a finite rates [2]. Bejan [3] reported the use of some of the basic FTT methodology in engineering since the early 1950s in the so-called *entropy generation method*. Today, FTT is considered a part of a wider field known as *thermodynamic optimization* [1]. A new concept of a saving function associated with thermal pollution was introduced by Angulo-Brown [4] and improved by Yan Z. [5]. The proposed ecological criterion is defined as:

$$E = \dot{W} - T_L \dot{S}_{gen} \tag{1}$$

where T_L is the heat sink temperature and $\dot{S}_{gen}$ is the entropy generation rate.

Yan proposed that is more reasonable to use:

$$E = \dot{W} - T_0 \dot{S}_{gen} \tag{2}$$

when the cold reservoir temperature is not equal to the environmental temperature.

The ecological optimization of an irreversible Rankine heat engine established that the heat engine operates under conditions between maximum power output point and maximum efficiency point.

In this paper, a steady-flow irreversible Rankine heat engine is presented. The irreversibilities are due to finite thermal conductance between the working fluid and the reservoirs and internal irreversibility. The heat engine produces power by converting a fraction of the received heat from the heating fluid (source of energy), the remaining heat being rejected to the cooling fluid (heat sink).

2. Analysis

In figure 1 is shown the schematic diagram, where $\dot{Q}_1$ is the heat rate absorbed from the hot fluid, $\dot{Q}_2$ is the heat rate released to the cold fluid by the working fluid and $\dot{W}$ is the generated power.

The temperature-entropy diagram of the irreversible Rankine cycle is shown in figure 2. The irreversible Rankine cycle consists of the following processes:

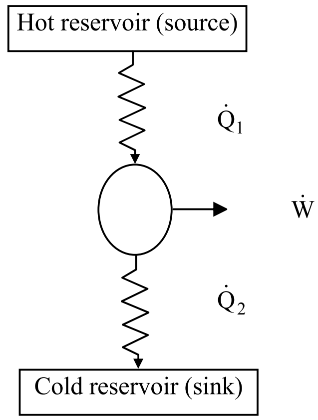

Fig. 1. *Schematic diagram of a Rankine heat engine*

1-2 irreversible compression of water in a pump;

2-3 constant pressure heat addition in a steam generator;

3-4 irreversible expansion of the superheated steam in a turbine;

4-1 constant pressure heat rejection in a condenser.

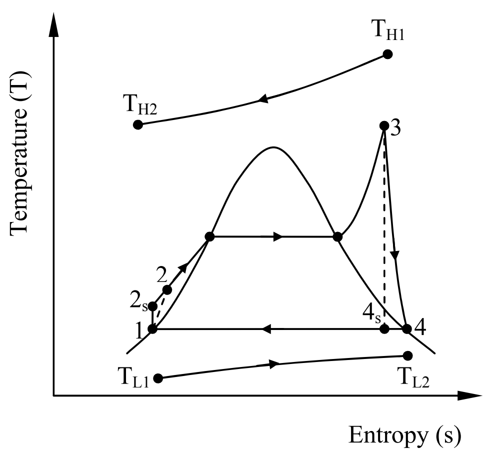

Fig. 2. *T-s diagram of a simple irreversible Rankine cycle.*

The irreversible Rankine cycle (1-2-3-4-1) can be modified to an irreversible Carnot cycle using an entropic average temperature in order to perform the mathematical model. The modified Rankine engine becomes a Carnot engine which operates between the entropic average temperatures T_1 and T_2 defined as follows:

$$T_1 = \frac{H_4 - H_1}{S_4 - S_1}$$
$$T_2 = \frac{H_3 - H_2}{S_3 - S_2} \tag{3}$$

where H_i, S_i (i=1, 2, 3, 4) denote the enthalpy and entropy of working fluid. The modified Rankine heat engine, including the irreversibilities of finite rate heat transfer between the heat engine and its reservoirs and internal dissipations of the working fluid, is shown in figure 3.

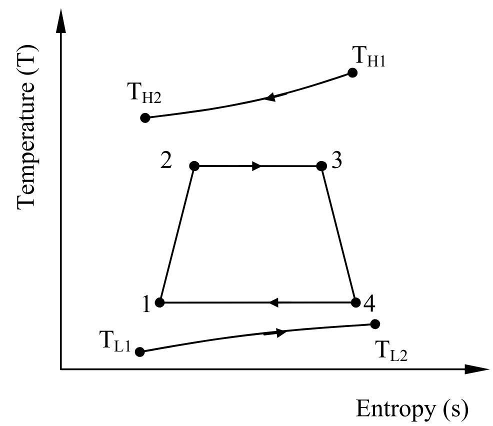

Fig. 3. *T-s diagram of a modified Rankine cycle*

We assume that the heat conductances ($U_H \cdot A_H$, $U_L \cdot A_L$) and the heat capacitances ($\dot{m}_H c_{pH}, \dot{m}_L c_{pL}$) are changing and the inlet temperatures of the heating fluid and cooling fluid (T_{H1}, T_{L1}) are fixed.

Considering a linear law for the heat transfer, we can write:

$$\dot{Q}_1 = U_H A_H \Delta T_{LMTD,H} = \\ = \dot{m}_H c_{pH} (T_{H1} - T_{H2}) = H_3 - H_2 \tag{4}$$

$$\dot{Q}_2 = U_L A_L \Delta T_{LMTD,L} = \\ = \dot{m}_L c_{pL} (T_{L2} - T_{L1}) = H_4 - H_1 \tag{5}$$

where: A_H, A_L - the total heat exchanger area of the hot heat exchanger and of the cold heat exchanger, respectively (kW/m^2K);

U_H, U_L – the overall heat transfer coefficient the hot heat exchanger and of the cold heat exchanger, respectively (m^2);

c_{pH}, c_{pL} – specific heat at constant pressure of the heating fluid and cooling fluid, respectively ($kJ/kg \cdot K$);

$$\Delta T_{LMTD,H} = \frac{(T_{H1} - T_2) - (T_{H2} - T_2)}{\ln \frac{T_{H1} - T_2}{T_{H2} - T_2}}$$ (log mean temperature-difference between the hot reservoir and the heat engine);

$$\Delta T_{LMTD,L} = \frac{(T_1 - T_{L1}) - (T_1 - T_{L2})}{\ln \frac{T_1 - T_{L1}}{T_1 - T_{L2}}}$$ (log mean temperature-difference between the heat engine and the cold reservoir).

The net power output of the Rankine heat engine is:

$$\dot{W} = \dot{Q}_1 - \dot{Q}_2 \tag{6}$$

Using equations (4) and (5) we can write:

$$T_{H2} = T_2 + (T_{H1} - T_2) e^{-NTU_H} \tag{7}$$

$$T_{L2} = T_1 - (T_1 - T_{L1}) e^{-NTU_L} \tag{8}$$

where: $NTU_H = \frac{U_H A_H}{\dot{m}_H c_{pH}}$ (number of transfer units for the hot heat exchanger);

$NTU_L = \frac{U_L A_L}{\dot{m}_L c_{pL}}$ (number of transfer units for the cold heat exchanger).

In order to include the effect of the internal dissipations of the working fluid on the performance of the heat engine it was introduced a parameter [6]

$$R = \frac{\Delta S_2}{\Delta S_1} \tag{9}$$

which characterizes the degree of internal irreversibility.

Applying the second law of thermodynamics to the irreversible Carnot cycle it can be written:

$$\frac{\dot{Q}_2}{T_1} - \frac{\dot{Q}_1}{T_2} > 0 \tag{10}$$

Using equation (9) the inequality in equation (10) becomes:

$$\frac{\dot{Q}_2}{T_1} - R\frac{\dot{Q}_1}{T_2} = 0 \tag{11}$$

The ecological function can be expressed as:

$$E = \dot{W} - T_0\left(\frac{\dot{Q}_L}{T_{L1}} - \frac{\dot{Q}_H}{T_{H1}}\right) \tag{12}$$

With equations (7) and (8) the net power output becomes:

$$\begin{aligned} \dot{W} = {} & \dot{m}_H c_{pH}(T_{H1} - T_2)(1 - e^{-NTU_H}) - \\ & - \dot{m}_L c_{pL}(T_1 - T_{L1})(1 - e^{-NTU_L}) \end{aligned} \tag{13}$$

From equation (13) we get:

$$T_1 = T_{L1} + \frac{a\dot{C}_H (T_{H1} - T_2)}{b\dot{C}_L} \tag{14}$$

where: $\dot{C}_H, \dot{C}_L$ - thermal capacitance rate of the heating fluid and cooling fluid, respectively, W/(kg K);

$$a = 1 - e^{-NTU_H}\ ;\ b = 1 - e^{-NTU_L} \tag{15}$$ (7)

Substitution of equation (14) into equation (11) yields

$$T_2 = \frac{-n_1\dot{W} + n_2 \pm \left[\left(n_1\dot{W} - n_2\right)^2 - 4m(-p_1\dot{W} + p_2)\right]^{1/2}}{2m} \tag{16}$$

where:

$$\begin{aligned} n_1 &= a\dot{C}_H + b\dot{C}_L \\ n_2 &= ab\dot{C}_H\dot{C}_L T_{H1} + ab\dot{C}_H\dot{C}_L T_{L1} + 2a^2\dot{C}_H^2 T_{H1} \\ p_1 &= a\dot{C}_H T_{H1}\ ; p_2 = ab\dot{C}_L\dot{C}_H T_{H1} T_{L1} + a^2\dot{C}_H^2 T_{H1}^2 \\ m &= a^2\dot{C}_H^2 + ab\dot{C}_H\dot{C}_L \end{aligned} \tag{17}$$

Using equation (16) the ecological function can be expressed as:

$$E = r\dot{W} + sT_2 - q \tag{18}$$

where:

$$\begin{aligned} r &= 1 + \frac{T_0}{T_{L1}}; s = a\dot{C}_H T_0\left(\frac{1}{T_{L1}} - \frac{1}{T_{H1}}\right) \\ q &= a\dot{C}_H T_0\left(\frac{T_{H1}}{T_{L1}} - 1\right) \end{aligned} \tag{19}$$

To maximize *E* with respect to $\dot{W}$ we take $\frac{\partial E}{\partial \dot{W}} = 0$ which yields

$$r + s\frac{\partial T_2}{\partial \dot{W}} = 0 \tag{20}$$

Solving equation (20) we get:

$$\dot{W}_{opt} = \frac{-A_2 \pm (A_2^2 - 4A_1A_3)^{1/2}}{2A_1} \qquad (21)$$

where:

$$A_1 = B_1^2 - B_2^2$$

$$B_1 = 1 + \frac{T_0}{T_{L1}} - a\dot{C}_H T_0 \frac{n_1}{2m}\left(\frac{1}{T_{L1}} - \frac{1}{T_{H1}}\right)$$

$$B_2 = a\dot{C}_H T_0 \frac{1}{2m}\left(\frac{1}{T_{L1}} - \frac{1}{T_{H1}}\right)$$

$$A_2 = 2n_1^2 + 4n_1mp_1B_2^2 - 2n_1n_2B_1^2 - 4mp_1B_1^2$$

$$A_3 = n_2^2B_1^2 - 4mp_2B_1^2 - n_1^2n_2^2B_2^2 - 4m^2p_1^2B_2^2 - 4n_1n_2mp_1B_2^2$$

From equation (16) and (21) we obtain T_{2opt} and then we can calculate T_{1opt}, $\dot{Q}_{1,opt}$ and $\dot{Q}_{2,opt}$.

The thermal efficiency corresponding to the maximum ecological function is given by

$$\eta_{opt} = \frac{\dot{W}_{opt}}{\dot{Q}_{1,opt}} \qquad (22)$$

3. Results and discussion

In figure 4, the variation of dimensionless ecological function $E/(\dot{C}_H T_{H1})$ with the dimensionless power output $\dot{W}/(\dot{C}_H T_{H1})$ for an irreversible Rankine cycle is shown.

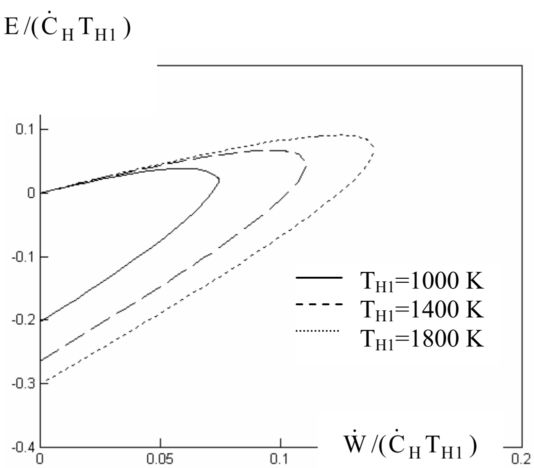

Fig. 4. *Ecological function versus power output for $U_HA_H = U_LA_L$ =1 kW/K, R=1.2, $\dot{C}_H = \dot{C}_L = 0.2$ kW/K*

It can be seen that the power output at maximum ecological function is lower than the maximum power that can be attained and it increases with the inlet temperature of the heating fluid. For a given power output correspond two values of ecological function. Obviously it will be chosen the higher values of ecological function in the design of an irreversible Rankine heat engine.

To see the effect of the thermal capacitance rate of the heating fluid on the variation of ecological function with power output it was drawn figure 5. The maximum of power output increases with thermal capacitance rate of the heating fluid or cooling fluid.

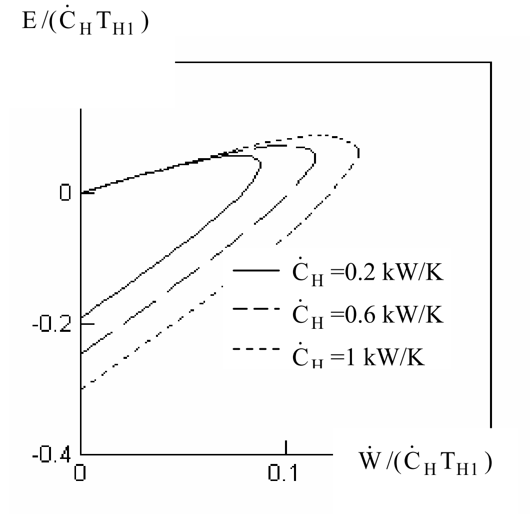

Fig. 5. *Ecological function versus power output for $U_HA_H = U_LA_L$ =1 kW/K, R=1.2 and T_{H1}=1800K*

In figure 6, the effect of the heat conductances of heating fluid U_HA_H on the variation of ecological function with power output is shown. As it is expected, the maximum of power output increases with heat conductance of the heating fluid or cooling fluid.

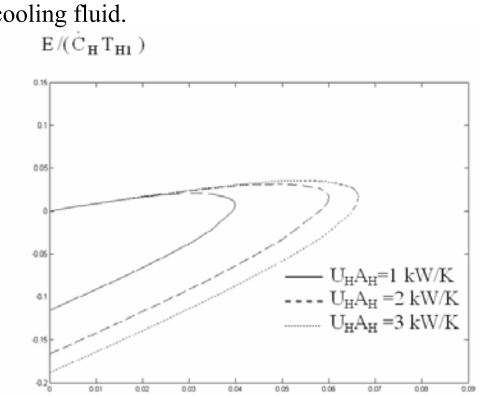

Fig. 6. *Ecological function versus power output for $\dot{C}_H = \dot{C}_L = 0.2$ kW/K, R=1.2 and T_{H1}=1800K*

The variation of ecological function with thermal efficiency of an irreversible Rankine heat engine is depicted in figure 7. It can be observed that the maximum ecological function point and the maximum thermal efficiency point exist. The thermal efficiency corresponding to the maximum ecological function is lower than the maximum thermal efficiency that can be achieved.

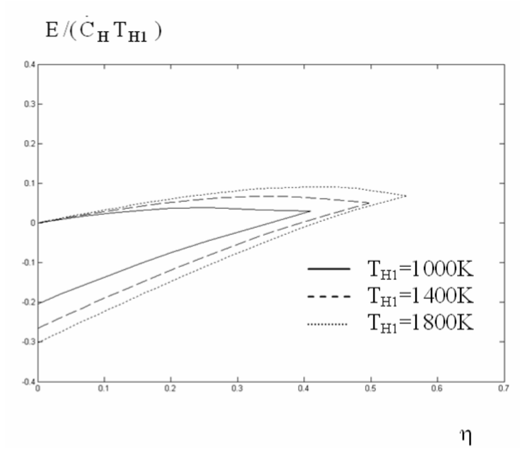

Fig. 7. *Ecological function versus thermal efficiency for $U_H A_H = U_L A_L$ =1 kW/K, R=1.2, $\dot{C}_H = \dot{C}_L = 0.2$ kW/K*

Figures 8 and 9 illustrate the variation of ecological function with power output and thermal efficiency, respectively of an irreversible Rankine heat engine for different values of internal irreversibility degree *R*.

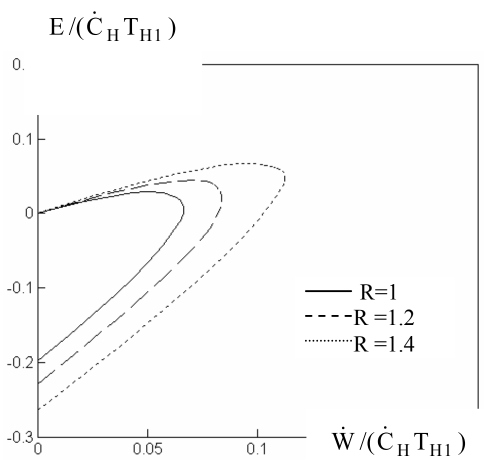

Fig. 8. *Ecological function versus power output for $\dot{C}_H = \dot{C}_L = 0.2$ kW/K, $U_H A_H = U_L A_L$ =1 kW/K, and T_{H1}=1400K*

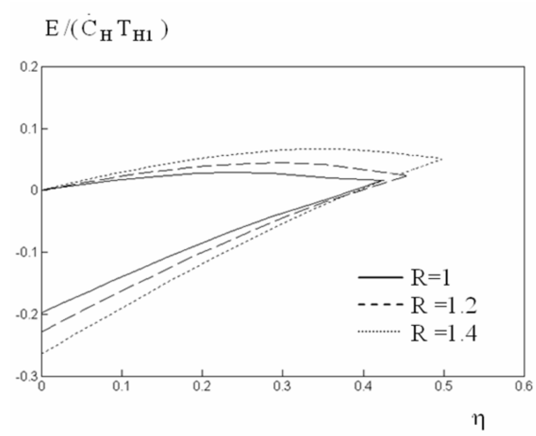

Fig. 9. *Ecological function versus thermal efficiency for $\dot{C}_H = \dot{C}_L = 0.2$ kW/K, $U_H A_H = U_L A_L$ =1 kW/K, and T_{H1}=1400K*

The optimal power output and the optimal thermal efficiency which correspond to the maximum ecological function decrease with the increase of *R* having the maximum value for *R*=1 (endoreversibil conditions).

4. Conclusion

Ecological optimization using finite-time thermodynamics of an irreversible Rankine heat engine has been performed. The thermodynamic irreversibilities due to finite thermal conductance between the working fluid and the reservoir and internal irreversibility have been considered. The maximum value of ecological function increases with inlet temperature of heating fluid, heat capacitance rates and heat conductances of heating and cooling fluids. The maximum value of ecological function decreases with the increase of internal irreversibility degree.

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Analiza ecologică bazată pe termodinamica în timp finit a unei maşini termice ireversibile Rankine

— Rezumat —

În lucrare sunt prezentate rezultatele studiului realizat pentru optimizarea ecologică a unei maşini termice ce evoluează ireversibil după un ciclu Rankine. Pentru a determina maximum funcției ecologice au fost folosite principiile termodinamicii în timp finit. Optimizarea ecologică oferă un compromis între condițiile de funcționare ce maximizează puterea generată şi condițiile de funcționare ce minimizează poluarea termică. Ireversibilitățile termodinamice se datorează conductanței termice finite dintre fluidul de lucru şi sursele de căldură precum şi disipării interne a fluidului de lucru (curgere cu frecare, pierdere de căldură etc.). Pentru a obține o valoare mai mare pentru funcția ecologică este necesar ca diferența dintre temperaturile celor două surse de căldură să fie cât mai mare. A fost studiat efectul conductanțelor termice, al capacitanțelor termice şi al gradului de ireversibilitate internă asupra valorii maxime a funcției ecologice şi parametrilor