## FE Simulation and Response Surface Methodology for Optimization of Tube Bending Process

PhD. eng. Lucian LĂZĂRESCU Technical University of Cluj-Napoca

## ABSTRACT

In this paper Finite Element Simulation and Response Surface Methodology (RSM) were used in order optimize the tube bending process. In a first stage, the amount of cross section ovality and wall thinning of bent tubes are determined by using the Finite Element Simulation. In the second stage, on the basis of the FE Simulation results, RSM was used to develop two empirical models able to describe the relationship between the cross section ovality and wall thinning, respectively and two independent variables: relative bending radius (R/D) and relative tube diameter (D/t). The models were tested using the ANOVA method, which demonstrated that the solutions fit sufficiently precise the results obtained by FE Simulation. The statistical models were used to establish an optimal technological window for the tube bending process in order to obtain products with an acceptable quality.

**KEYWORDS**: Tube bending, FE Simulation, Response Surface Methodology, Optimization.

## **1. Introduction**

The metallic bent tubes are widely used in the automotive, aerospace, aircraft, petroleum industries, and various other industries due to its satisfying high strength/weight ratio requirement [1]. The bending of tubes can be done by several different methods such as ram bending, press bending, compression bending, rotary draw bending, and the like. Among these methods, the rotary draw bending method is the most versatile, cost-effective, and precise method. In the actual conditions of the steadily rising demands of the customers concerning the higher quality of bent tubes, concomitantly with the diminution of manufacturing costs, the manufacturers of bent tubes are constrained to continuously improve their forming techniques.

For assessment the quality of bent tubes, measurable characteristics (determined by measurements) or attributive characteristics (classification of the parts into "good" or "bad") can be used. A bent tube can be for example considered "good" when in the bent area there is no incidence of cross section, do not appear wrinkles, cracks, marks etc. The group of measurable characteristics of quality includes: the cross section ovality, excessive wall thinning, spring back, the error of distance between bending, plane bending error, surface roughness etc. [2]. Veerappan and Shanmugam [3] have observed that the cross section ovality and the wall thinning should be considered together in order to determine the acceptability of a bent tube in terms of accuracy. The strength of the bent tube against the internal pressure is even greater as the cross section is closer to the circular shape. On the other hand the wall thinning in the extrados causes also the weakening of the strength. This thinning has undesired consequences, because the inner wall in the outside of the curvature is subjected to wear by friction during the employment, because of action of the product that flows through the tube.

In the design phase of tube bending process it is important to know the values of geometrical parameters: relative bending radius R/D (R is the bending radius and D is the tube original outside diameter) and relative tube diameter D/t (t is the original tube wall thickness) which assures that a certain quality of the obtained products.

Up to now, few papers have dealt with the optimization of the quality of tubes during rotary draw bending process. Li et al. [4] has studied analytically the bending limit of tubes and has establish the variation of minimum bending radius R

as function of tube diameter D and tube wall thickness t. Veerappan and Shanmugam [3] have presented a mathematical relationship among the pressure ratio, ovality, thinning, tube ratio and bend ratio and the variation of constant pressure ratio with shape imperfection were analysed. In the paper [5], a search algorithm of the forming limits is put forward based on a 3D elastic-plastic finite element model and a wrinkling energy prediction model for the bending processes under axial compression loading. Based on this algorithm, the forming limits of the different size tubes are obtained, and the roles of the process parameter combinations in enabling the limit bending processes are also revealed.

The aim of this paper is to develop a new approach in order to determine the optimum values of geometrical parameters (R/D and D/t), so as to not exceed a certain values of two quality characteristics: cross section ovality  $\Psi$  and wall thinning  $\xi$ . This approach consists in determination of two empirical models able to describe the relationship between quality characteristics and geometric parameters. Then the empirical models were used in order to establish a technological window for the optimal tube bending. An analysis of the relationship between geometrical parameters and variation of the quality characteristics during bending are also discussed using the results obtained by empirical models.

## 2. Tube Bending Process

#### 2.1 Rotary draw bending of tubes

Figure 1.a) shows the principle of rotary draw tube bending of tubes. Using a rotary-draw tube bending machine, a rotating bending die forms the tube to the radius of the bending die. A clamp die secures the tube to the bending die. The tube bending machine rotates the bending die to the desired bend angle while a sliding pressure die forces the tube to conform to the die radius. A wiper die is often placed behind the pressure die to prevent wrinkling and buckling of the part. After bending, the operator releases the clamp die and then removes the bent tube from the machine.

# 2.2 Characteristics for assessment of bent tube quality

In order to assess the effects of various process parameters on the cross section quality of bent tubes, a quality factor is defined. This factor, called the cross section ovality ( $\Psi$ ), is defined as the percentage of cross section deviation of a bent tube from its original circularity:

$$\psi = \frac{D_{\text{max}} - D_{\text{min}}}{D} \times 100 \%, \qquad (1)$$

where:  $D_{\text{max}}$  is the maximum tube diameter after bending,  $D_{\text{min}}$  - the minimum tube diameter after bending and D - the initial value of tube outside diameter (Fig. 1.b).



Fig. 1. Rotary draw tube bending: a)-principle of method; b)- cross section distortion

To assess the variation of wall thinning of each cross section along the bending part, the wall thinning degree ( $\xi$ ) is defined as:

$$\xi = \frac{t - t_{\min}}{t} \times 100 \%, \qquad (2)$$

where t is the initial value of wall thickness, and  $t_{min}$  is the minimum wall thickness along the outside of the bend (Fig. 1).

## 3. FE Modelling of Tube Bending

In this study an elasto-plastic three-dimensional finite-elements model of tube bending process is developed using a commercially available explicit finite element code eta/Dynaform.

#### 3.1 Material properties of tube

To obtain the mechanical properties of the steel tube a tensile test of straight tubular specimens was performed. The mechanical properties of the tube are shown in Table 1. In order to describe the material behaviour during simulation, the power law described by Hollomon's equation was used:

$$\sigma = K \cdot \varepsilon^n \tag{3}$$

 Table 1. Mechanical properties of tube

Material: OLT 45 SR EN 10216-2:2008	Value
Ultimate tensile strength, $\sigma_b$ (MPa)	450
Initial yield stress, $\sigma_v$ (MPa)	260
Total elongation, $\delta$ (%)	21
Hardening exponent, <i>n</i>	0.39
Strength coefficient, K (MPa)	698
Young's modulus E (MPa)	$2.1 \times 10^{5}$
Poisson's ratio, $\gamma$	0.3
Mass density, $\rho$ (kg/m <sup>3</sup> )	7850

#### 3.2 Finite element model of tube bending

The rotary draw bender includes five tools, namely, bending die, clamp die, insert die, pressure die and wiper die (shown in Fig.2). The 3D-FE model of rotary-draw bending process of cold rolled steel OLT 45 circular tube and meshing are shown in Figure 3. During the bending process, the tube is deformable so it is defined as deformable body. Shell elements of type BELYTSCHO-TSAY were used to model the tube geometry with 5 points of integration. And the dies are not deformable so they are defined as discrete rigid bodies. The contact between various pairs of surfaces (bending die-tube and pressure dietube) defined was using the FORMING\_SURFACE\_TO\_SURFACE contact option, which allows sliding between these surfaces with a Coulomb friction model. The implemented friction coefficient was 0.125.

## 3.3 Experimental validation of FE model

In order to validate the reliability of FE model a rotary draw hidraulic bending machine is used in order to achieve the experiments. The validation of FE model was caried out in term of variation of cross section ovality as function of bending radius. For this purpose three specimens made from cold rolled steel OLT 45 with outside diameter D=32 mm and original wall thickness t=2.8mm, were bent at bending angle  $\theta$ =120° and at different bending radii: R=100 mm; R=80 mm and R=58 mm. After bending, the tubes were cut in so-called critical cross section at an angular position of 60°, measured from the tangent line (Fig.1.a) and the diameters  $(D_{\text{max}} \text{ and } D_{\text{min}})$  were measured. The cross section ovality were calculed using Equation (1). The experimental cross section ovality is 8.30%; 12.10% and 14.89% respectively. Figure 4 and Figure 5 compare the simulation result with the experimental result obtained under the condition described above. By comparison of cross

section ovality between FE Simulation and experimental results, it is found that the cross section ovality is nearly the same between simulation and experimental results. Therefore the developed FE model can simulate the rotary draw bending process well and can be used to study the the quality of tubes during bending process reliably.



Fig. 2. Geometry model for rotary draw bending of tubes



Fig. 3. Explicit FE model for the rotary draw bending process: a)-initial position; b)-during bending

Using FE- simulation, the cross section ovality and maximal wall thinning, were determined as a function of relative bending radius (R/D) and the relative tube diameter (D/t). These FE simulations were conducted under the planning of experiments in Table 3 and the results were used to establish two empirical models, as it is described in section 4.



Fig. 4. Photos tubes bent at different bending radii (D=32 mm and t=2.8 mm)

## 4. Response Surface Methodology

The response surface methodology (RSM) is a collection of statistical and mathematical techniques used to examine the relationship between one or more response variables and a set of quantitative experimental variables or factors.

## 4.1 Theoretical formulation

Response surface modelling postulates a model of the form:

$$y(\mathbf{x}) = f(\mathbf{x}) + \varepsilon \tag{4}$$

where  $y(\mathbf{x})$  is the unknown function of interest,  $f(\mathbf{x})$  is a known polynomial function of  $\mathbf{x}$ , and  $\varepsilon$  is random error which is assumed to be normally distributed with mean zero and variance  $\sigma^2$ . The individual errors,  $\varepsilon_i$ , at each observation are also assumed to be independent and identically distributed. The polynomial function,  $f(\mathbf{x})$ , used to approximate  $y(\mathbf{x})$  is typically a low order polynomial which in this paper is assumed to be quadratic, Eq. (5).

$$\hat{y} = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_i \sum_{j>i} \beta_{ij} x_i x_j \quad (5)$$

The parameters,  $\beta_0$ ,  $\beta_i$ ,  $\beta_{ii}$ , and  $\beta_{ij}$ , of the polynomial in Eq. (5) are determined through least squares regression which minimizes the sum of the squares of the deviations of the predicted values,  $\hat{y}(\mathbf{x})$ , from the actual values,  $y(\mathbf{x})$ . The coefficients

of Eq. (5) used to fit the model can be found using the least square regression given by Eq. (6):

$$\beta = [XX]^{-1}X\mathbf{y}, \tag{6}$$

where X is the design matrix of sample data points, X is its transpose, and y is a column vector containing the values of the response at each sample point.



Fig. 5. Bent tubes obtained from FE Simulation (D=32 mm and t=2.8 mm)

#### 4.2 Experimental design

Design of experiment is a technique for setting an efficient point parameter. A well designed series of experiments can substantially reduce the total number of experiments. The most widely used classes of response surface design are the full factorial design. orthogonal design, central composite design. In this paper a central composite design (CCD) with two factors was used as is shown in Figure 6. The CCD contains an imbedded factorial design with centre points that is augmented with a group of "star points" that allow the estimation of curvature. If the distance from the centre of the design space to a factorial point is  $\pm 1$  for each factor, the distance from the centre of the design space to a star point is  $\pm \alpha$  with  $|\alpha| > 1$ , presented in Table 2. The value of  $(\alpha)$ , in case of two factors is 1.414.



Fig. 6. Two-Variable Rotatable CCD

Relative bending radius R/D ( $x_1$ ) and relative tube diameter D/t ( $x_2$ ) were chosen for independent variables. Cross section ovality ( $\Psi$ ) and wall thinning degree ( $\xi$ ) were selected as the response for the combination of the independent variables given in

Independent	Symbol	Code	d valu	es (α =	= 1,414	)
variables	Symbol	-α	-1	0	1	$+\alpha$
(R/D)	$x_1$	1.75	2.01	2.63	3.24	3.5
(D/t)	$x_2$	8	9.17	12	14.83	16

**Table 2.** Independent variables and level values

Table 3. Experimental plan for the codea	l values in
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Table 2 and FE Simulation results	

Run	Coded independent variables		Response variable		
Kull	$x_1$	$x_2$	Ovality	Thinning	
	(R/D)	(D/t)	Ψ[%]	ξ[%]	
1	-1	-1	15.26	11.10	
2	1	-1	7.28	7.73	
3	-1	1	19.80	10.55	
4	1	1	9.94	7.62	
5	-α	0	18.96	11.87	
6	α	0	7.94	7.20	
7	0	-α	8.77	9.08	
8	0	α	13.65	8.77	
9	0	0	11.16	9.01	

Analysis of the model proposed for the observed data, and calculation of its coefficients, were carried out using the Design Expert Software (version 7.1 trial Statease Inc., Silicon Valley, CA, USA) [6].

## 5. Results and Discussion

## 5.1 Model Fitting

## 5.1.1 The model of cross section ovality

Equation (7) shows the dependence of the cross section ovality ( $\Psi$ ) on the relative bending radius  $R/D(x_1)$  and relative diameter of the tube  $D/t(x_2)$ . The coefficients of the equation were obtained by the multiple regression analysis of the FE Simulation results. The following quadratic model approximates the FE Simulation data:

$$\psi(x_1, x_2) = 40.66 - 21.76x_1 + +0.69x_2 - 0.26x_1x + 3.47x_1^2 +,$$
(7)  
+0.02x\_2<sup>2</sup>

In order to know how well a regression model approximates the real data points, a statistical measure, namely *R*-squared is introduced. The closer the value of  $R^2$  to unity, the better the empirical model fits the actual data. Multiple regression analysis results were  $R^2$ =0.9900 of the quadratic model, indicating a high degree of correlation between the FE Simulation values presented in Table 3 and the predicted values obtained from the model.

Statistical testing of the empirical model has been done by the Fisher's statistical test for Analysis Of Variance – ANOVA. Table 4 shows the ANOVA test applied to the individual coefficients in the model, to test their significance.

**Table 4.** ANOVA for quadratic model of cross sectionovality

Source	SS	DF	MS	F-value	<i>P</i> -value	
Model	172.52	5	34.50	59.41	0.0034	
$x_1$	139.65	1	139.65	240.48	0.0006	
$x_2$	24.86	1	24.86	42.80	0.0073	
$x_1 x_2$	0.88	1	0.88	1.52	0.3052	
$x_1^2$	5.15	1	5.15	8.86	0.0587	
$x_2^2$	0.13	1	0.13	0.22	0.6704	
Residual	1.74	3	0.58			
Total	174.26	8				
$R^2 = 0.999$ ; SS-sum of square;						
DF-degree of freedom; MS-Mean Square						

The F-value is the ratio of the mean square due to regression to the mean square due to residual. The model *F*-value of 59.41 implies that the model is significant. The calculated F-value of the model should be greater than the tabulated value for a good model. F-value is compared to the tabulated value  $F(v_1, v_2)$ , where  $v_1$  is the degree of freedom of the model and  $v_2$  is the degree of freedom of the residual. In our case, for a reliance threshold of 0.01, we find  $F_{0.01}(5,3)=5.31$ . Therefore the calculated F-value is greater than the tabulated value and the null hypothesis is rejected. Values of "Prob>F" less than 0.0500 indicate that the model terms are significant. In this case,  $x_1$  and  $x_2$  are significant model terms and the quadratic term  $(x_1^2)$  also had highly significant effect. Values greater than 0.1000 indicate the model terms are not significant.

## 5.1.2 The model of wall thinning degree

Equation (8) shows the dependence of the wall thinning degree  $\xi$  on the relative bending radius R/D ( $x_1$ ) and relative diameter of the tube D/t ( $x_2$ ).

$$\xi(x_1, x_2) = 23.6 - 7.35x_1 - 0.21x_2$$
  
-0.06x<sub>1</sub>x<sub>2</sub> + 0.76x<sub>1</sub><sup>2</sup> - 4.68 \cdot 10<sup>-3</sup> x<sub>2</sub><sup>2</sup>, (8)

The determination coefficient  $R^2=0.9991$  of the quadratic model, indicating a high degree of correlation between the FE Simulation values presented in Table 3 and the predicted values obtained from the model described by Equation (8).

Table 5 shows the ANOVA test applied to the individual coefficients in the model (Eq.(8)), to test their significance. The model *F*-value of 694.45 implies that the model is significant. There is only a 0.01% chance that a "Model *F*-Value" this large could occur due to noise. Values of "Prob>*F*" less than 0.0500 indicate model terms are significant. In

this case  $x_1, x_2, x_1^2$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

 Table 5. ANOVA for quadratic model of wall

 thinning degree

Source	SS	DF	MS	<i>F</i> -value	P-value	
Model	21.43	5	4.29	649.45	0.0001	
$x_1$	20.82	1	20.82	3372.06	0.0001	
$x_2$	0.15	1	0.15	24.43	0.0159	
$x_1 x_2$	0.004	1	0.048	7.84	0.0678	
$x_1^2$	0.21	1	0.21	33.72	0.0102	
$x_2^2$	$4.09 \cdot 10^{-3}$	1	$4.09 \cdot 10^{-3}$	0.66	0.4752	
Residual	1.74	3	$6.17 \cdot 10^{-3}$			
Total	174.26	8				
$R^2$ =0.9991; SS-sum of square;						
DF-degree of freedom; MS-Mean Square						

## 5.2 **Response surfaces**

The 3D response surface given by Equation (7) is shown in Figure 7 and the surface given by Equation (8) is shown in Figure 8. These surfaces were obtained by Design Expert Software.



Fig. 7. The 3D response surface of the cross section ovality



Fig. 8. The 3D response surface of wall thinning

The plots depict the dependence between quality characteristics (i.e. cross section ovality and wall thinning) and relative bending radius (D/R) and the relative tube diameter (D/t), respectively.

A contour plot is a graphical technique for representing a 3-dimensional surface by plotting constant z slices, called contours, on a 2-dimensional format. Equation (7) is plotted in Figure 9 and the Equation (8) is plotted in Figure 10 as a contour of the response surface. These contours were also obtained by Design Expert Software. These curves have a special significance from technological point of view, because based on these, a technological window will be determined for the two geometrical parameters, that ensure the obtaining of bent tubes with an acceptable quality.

For example if we intend to obtain bent tubes with a cross section ovality smaller than 9.57%, it is necessary to choose values of the geometrical parameters (R/D) and (D/t) from the right side of the constant curve of ovality of 9.57%, Figure 9.







Fig. 10. Wall thinning contours plot

# 5.3 Effect of R/D and D/t on the quality of tubes

The relative bending radius R/D and relative tube diameter are important process parameters affecting the quality of bent tubes. Using the Equation (7) and Equation (8), the cross section ovality ( $\Psi$ ) and wall thinning degree ( $\xi$ ) were determined as a function of the relative bending radius (Fig. 11) and as function of relative tube diameter (Fig.12). As it can be seen, from Figure 11, both cross section ovality and minimal wall thinning decrease with increasing of relative bending radius. At the increase of relative bending radius from 1.86 to 3.5, a decrease of cross section ovality with 10% and of wall thinning degree with 4.12% was obtained. Figure 12 shows that the cross section ovality increases with increasing of relative tube diameter while the wall thinning degree decreases slightly with increasing of relative tube diameter. The variations of two quality characteristics depending on the relative bending radius and relative tube diameter can be seen also in the Figures (7 -10).



Fig.11. Influence of R/D on the tube quality factors



Fig.12. Influence of D/t on the tube quality factors

## 5.4 Graphical optimization of tube bending process

Next the problem that we intend to resolve is to establish the domain of values of the two independent variables  $x_1$  and  $x_2$  (i.e. R/D and D/t) so as the cross section ovality and wall thinning, respectively to not exceed a certain value. This problem will be solved using the Equations (7) and (8).

We intend to determine the graphic area of the values of the relative bending radius (R/D) and of the relative tube diameter (D/t) which ensures the obtaining of a cross section ovality ( $\Psi$ ) smaller than 10% and a wall thinning ( $\mathcal{E}$ ) under 8.40%. From mathematical point of view, the optimization problem can be written as follow: using Equations (7) and (8) we determine the area of independent variables:  $x_1 \in [1.75, 3]$  and  $x_2 \in [8, 15]$  so as:  $\Psi < 10\%$  and  $\xi < 8.40\%$ . In order to resolve this problem, the curves corresponding to the values  $\Psi$ =10% and  $\xi$ =8.40% are overlay plotted, Figure 13. The technological window for the tube bending process is at the right side of the curve corresponding to the ovality limit of 10% and also at right side of the curve corresponding to the wall thinning limit of 8.40%, Figure 13.



## 6. Conclusions

In this paper a new approach is proposed for optimization of tube bending process. This approach consists in the use of FE Simulation and Response Surface Methodology in order to establish a relationship between the cross section ovality and wall thinning degree, respectively and relative bending radius (R/D) and relative tube diameter (D/t). The result of the ANOVA test shows that the "fit" of the models to the FE Simulation data was statistically sufficiently precise.

Using the empirical models a technological window for the tube bending process limited by  $\Psi < 10\%$  and  $\xi < 8.40\%$  are determined. Therefore, the models are adequate to be used in the optimization of cross section ovality and wall thinning degree and, consequently, the proposed approach is considered to be suitable for industrial applications.

After the study of the influence of geometrical parameters it was found that both the cross section ovality and minimal wall thinning decrease with increasing of relative bending radius. On the other hand it was found that the cross section ovality increases with increasing of relative tube diameter while the wall thinning degree decreases slightly with increasing of relative tube diameter.

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#### Simularea cu elemente finite și metodologia suprafeței de răspuns pentru optimizarea procesului de îndoire a tuburilor

## Rezumat

În această lucrare este prezentată o nouă metodologie pentru optimizarea procesului de îndoire a tuburilor. Această metodologie constă în stabilirea unei ferestre tehnologice a parametrilor geometrici a procesului de îndoire (raza relativă de îndoire R/D și diametrul relativ al tubului D/t) în vederea obținerii unor tuburi îndoite cu caracteristici de calitate acceptabile. Caracteristicile pentru aprecierea calității tuburilor îndoite, considerate în această lucrare, sunt ovalitatea secțiunii transversale ( $\Psi$ ) și subtirea maximă a peretelui tubului ( $\zeta$ ). Într-o primă etapă, au fost determinate valorile celor două caracteristici de calitate cu ajutorul unui model de simulare numerică validat experimental, dezvoltat în programul comercial eta/Dynaform. În continuare, utilizând rezultatele simulărilor numerice, cu ajutorul Metodologiei Suprafeței de Răspuns au fost stabilite două modele empirice capabile să descrie legătura dintre parametrii geometrici (R/D și D/t) și caracterisicile de calitate ( $\Psi$  și  $\zeta$ ). Modele empirice au fost testate cu ajutorul metodei ANOVA, care a demonstrat că soluțiile găsite sunt în bună concordanță cu rezultatele obținute prin simulare cu elemente finite.

## FE Simulation und Response Surface Methodology für die Optimierung von Rohrbiegeprozess

#### Zusammenfassung

In dieser Arbeit wurden FE Simulation und Response Surface Methodology (RSM) eingesetzt, um die Biegeprozess von Rohren zur optimiren. Als erster Schritt wurden die Höhe der Querschnittovalität und Wandverdünnung von gebogenen Rohren mit Hilfe der Finite Elemente Simulation bestimmt. Als zweiter Schritt, auf der Grundlage der FE Simulationergebnisse wurde RSM verwendet, um zwei empirische Modelle zu entwickeln, die die Beziehung zwischen dem Querschnittovalität beziehungsweise Wandverdünnung und zwei unabhängigen Variablen: relative Biegeradius (R/D) und die relative Rohrdurchmesser (D/t) beschreiben können. Die Modelle wurden mit dem ANOVA-Methode geprüft, die gezeigt haben, dass die Ergebnisse von empirischen Modelle mit den Ergebnisse von FE Simulation hinreichend genau gepast sind. Die statistischen Modelle wurden verwendet, um eine optimale technologische Fenster für den Rohrbiegeprozess zu finden, um Produkte mit einem akzeptablen Qualität zu erhalten.