

## The Response Surface Method Applied to Deep Drawing with Combined Restraint

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### ABSTRACT

The deep drawing with combined restraint assures a greater degree of deformation in comparison with the conventional method of deformation. An important thickness variation appears during the deformation process. In the paper the method of the response surfaces for minimizing this variation has been applied. The response surface method considers the relation between the parameters of a process, in the present case, the die radius and the blank diameter and its characteristic answers as surfaces in the dimensional space of the variables, in this case, the thickness variation. An optimum value of the deforming parameters, is finally obtained.

**KEYWORDS:** deep drawing, response surface method, optimisation, numerical simulation

### 1. Introduction

The deep drawing with combined restraint is a particular process of deformation, in which the restraint of the blank takes place in two successive stages (figure1).

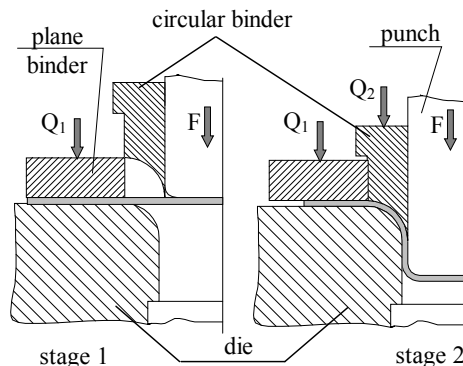


Fig. 1. The deep drawing with combined restraint [1]

First, the material is deformed with the blank restraint under the plane surface of the first binder, till it deforms along the die radius. Then, in the next stage, the process of deformation continues with the restraint of the blank on the plane zone using the first binder and on the die radius zone using another

binder (in this case a circular one). The presence of the second binder is a result of the die design which had in this case a higher radius die.

Some of the major advantages of the process are: the presence of the second binder leading to the increase of the possibility to obtain a deep drawing ratio  $m$  of about 0.42 for the first operation. [1, 2]. This means that the first two deep-drawing operations could be cumulated in only one operation, so the costs with the equipments and labour are reduced, for the first operation by 50%; the higher radius die leading to a smaller deep drawing force so the costs with energy are reduced; the durability of the die is increasing because the wear of the die is smaller as a result of the presence of a higher die radius.

A drawback of the method is the equipment design that becomes complicated if a press with simple or double action is used. This drawback limited the industrial application of the method [2].

Another problem which appears as a result of the deformation process is the variation of the thickness. Both the experimental and numerical simulations show that this variation is important, a higher material thickening appearing at the front of the part and a higher material thinning at the bottom of the part. So, it is necessary to optimize the process parameters for minimizing this variation. In what follows, the response surface method will be presented, having as objective the reduction rate on

the thickness variation of the pieces. The two process parameters considered in this paper are the die radius and the blank diameter.

## 2. The Response Surface Method

The response surface method considers the relation between the parameters of a process and its characteristic answers as surfaces in the dimensional space of the variables. In the experiments conducted relying on this method, the independent variables are fluctuated simultaneously taking a limited number of values and their principal effects and the first-class order, as well as the interactions between them separately determined.

The response surface method generally, covers the next steps: choosing the form and the complexity of the proposed mathematic model; programming the experiment; setting up the experimental conditions; carrying out the experiment; determination of the model coefficients; justifying the significance of the coefficients; establishing the intervals of confidence.

Generally, the mathematic modelling of a process or its given answer function takes into account the functional relation marked by the physical reality between the  $k$  parameters of the process as independent variables  $(x_1, x_2, \dots, x_k)$  and one of its characteristics as a dependent variable of response.

$$\eta = f(x_1, x_2, \dots, x_k) + \varepsilon \quad (1)$$

where: 1, 2 ... k represents the number in the factorial experiment. The terms  $x_k$  represent the level of the  $k$ th factor in the experiment. The function  $\eta$  is called the response surface. The residual  $\varepsilon$  measures the experimental error in the observation.

The geometric representation, in space, of the function  $\eta$ , with  $k+1$  dimension of the process variables will be a surface named response surface whose points have as coordinates correspondent values of the process parameters (figure 2).

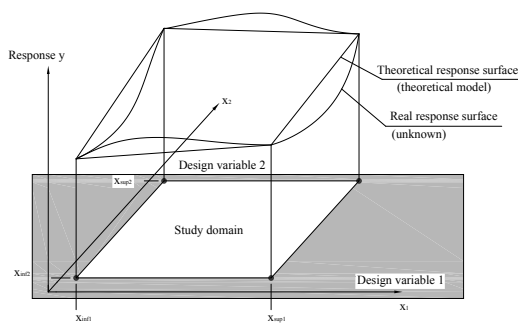


Fig. 2. Response surface geometry

It is necessary for the establishment of every process parameter variation level to know the respective domain according to the operation technological conditions. Knowing the domain of variation for each parameter, the centre of the experiment could be established, possibly corresponding to the origin around which the function of response, in Taylor series, had been developed. For the easiness of the coefficient determination and the statistic analysis fulfilment of the model, it is necessary that the natural variables and their level of variation could be codified.

Polynomial curve fitting equations normally exist both of first degree and second degree. They are also referred to as first order or the second order polynomials. The first order polynomials have the form:

$$y = b_0 + b_1x_1 + b_2x_2 + \dots + b_kx_k + \varepsilon \quad (2)$$

The second order polynomial known as the quadratic response surface has  $2-x$  variables, and takes the form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2 + \varepsilon \quad (3)$$

As the  $f$  function in equation (1) is unknown, it will be replaced with a correspondent polynomial expression and then the expression from the right member with the approximation model, becoming thus:

$$y = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i=1}^k b_{ii} x_i^2 + \sum_{i < j} b_{ij} x_i x_j \quad (4)$$

The deep drawing with combined restraint experiment is based on the second order design. In the current investigation, there are two  $x$  variables,  $x_1$ , and  $x_2$ , which correspond to the following independently controllable process parameters: radius of the die ( $R$ ), and the blank diameter ( $D$ ).

The experimental data that are necessary for the determination the process model are obtained carrying a certain number of experiments and measuring the correspondent answers. In the conditions of the surface adjustment of the experimental data, it is important to take into account both the number of the experiments and the number of experimental points as well as their placing in the experimental space. This problem it is tight related to the error measure and the complexity of the response surface.

The selected process parameters with their limit units and notations are given in table 1.

The dates for optimisation are obtained by simulation using the software Dynaform.

**Table 1. Process parameters and their limits**

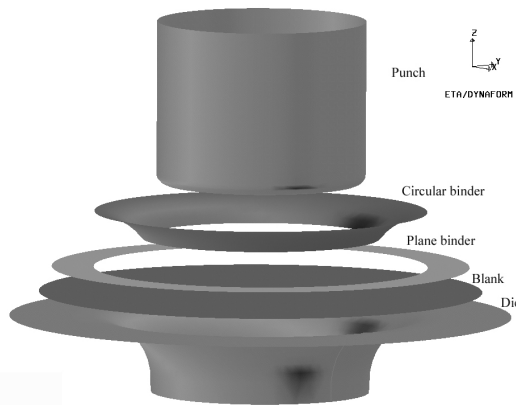
Parameters	Notations	Limits				
		-2	-1	0	1	2
Blank diameter (mm)	D	101	106	111	116	120
Die radius (mm)	R	15	16	17	18	19

### 3. Numerical Procedure

The simulation of the deep drawing process is carried out using the commercial software Dynaform. The Belytschko-Lin-Tsay shell element based on a combined co-rotational and velocity-strain formulation was chosen to analyze the elasto-plastic process with complex geometrical nonlinearity. The elements provide five integration points through the thickness of the sheet metal. The tooling was modelled as rigid surfaces. The investigations were based on a coefficient of friction equal to 0.1.

An expanded view of the tooling is presented in figure 3.

The material used in experiments was medium steel, with a thickness of 0.9 mm, chosen from the program material database, BH180, similar as properties to the real one. The mean properties of the material were: the yield stress of 196 MPa and the work hardening coefficient  $n$  of 0.19. The material was assumed to be anisotropic. The  $R$ -value at  $0^\circ$  was 1.65; at  $45^\circ$  was 1.25 and at  $90^\circ$  was 1.80.



**Fig. 3. Tooling used in simulation of deep drawing with combined restraint**

The yielding of the material was modelled using a power law, as:

$$\sigma = K \varepsilon^n \tag{5}$$

where  $K$  (MPa) is the material constant,  $K = 567$  MPa.

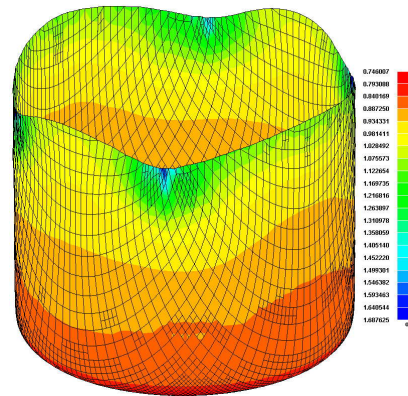
The punch speed was 5 mm/second.

The dimensions of the active elements were in accordance with the values presented in table 2.

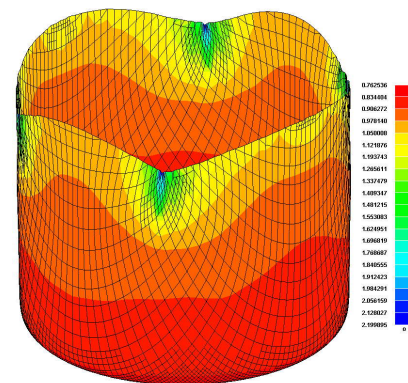
**Table 2. Main active elements dimensions**

Active element	Size (in mm)
Die diameter	52.25
Punch diameter	50
Radius die	Variable (15-20)

Figure 4 presents the thickness variations of the simulated samples for a deep drawing ratio of 0.49, corresponding to blanks diameters of 115 mm. The forms are quite the same as in the real case. The simulation results show an important variation of the thickness which increases with increasing the degree of deformation.



D=115 mm and R=15 mm



D=115 mm and R=20 mm

**Fig. 4. Thickness variations in deep drawing with combined restraint**

### 4. Results and Discussions

The response surface method applied to the deep drawing with combined restraint is used further to optimize the dimension of the circular blank and of the die radius so the thickness variation to be minimized.

So, the objective consists in reducing the thickness variation of the part in the final stage of the deep-drawing. The first step is to measure the thickness of parts for all the twenty-five simulations, along the height from the bottom to the end, in a number of equal points. The points were measured on the directions of 0 degree and 90 degree in rapport with the direction of lamination.

The objective function considered has the following formula:

$$f_0 = \sum_{i=1}^n \left| \frac{g_i - g_0}{g_0} \right| \tag{6}$$

where:  $n$  is the total number of measured points on the height;  $g_i$  – the value of the thickness in the point  $i$ ;  $g_0$  – the initial thickness of the blank,  $g_0=0,9$  mm.

The values of the objective function are presented in table 3.

**Table 3.** The objective functions of the studied process

R (Deep drawing die radius), [mm]	D (Blank diameter), [mm]	Objective functions	
		0 degrees	90 degrees
15	101	0.832	0.844
15	106	0.880	0.956
15	111	0.871	1.002
15	116	0.838	0.850
15	120	0.942	0.916
16	101	0.755	0.847
16	106	0.765	0.864
16	111	0.864	0.863
16	116	0.921	0.855
16	120	0.904	0.912
17	101	0.863	0.856
17	106	0.881	0.842
17	111	0.847	0.812
17	116	0.938	0.908
17	120	0.907	0.837
18	101	0.848	0.876
18	106	0.854	0.922
18	111	0.872	0.946
18	116	0.863	0.847
18	120	0.895	0.939
19	101	0.762	0.935
19	106	0.784	1.28
19	111	0.795	0.812
19	116	0.891	0.812
19	120	0.854	0.760

For applying the response surface method the Matlab program was used. By the implementation of the surface method for every face of the piece (at 0 degrees and at 90 degrees) we obtain 2 functions in 2 variables for which the minimum has to be obtained:

- for the direction of 0 degree:

$$y_1 = 0.8714 - x_1 \cdot 0.0086 + x_2 \cdot 0.0234 + x_1 \cdot x_2 \cdot 0.0013 + x_1^2 \cdot 0.0074 - x_2^2 \cdot 0.0002 \tag{7}$$

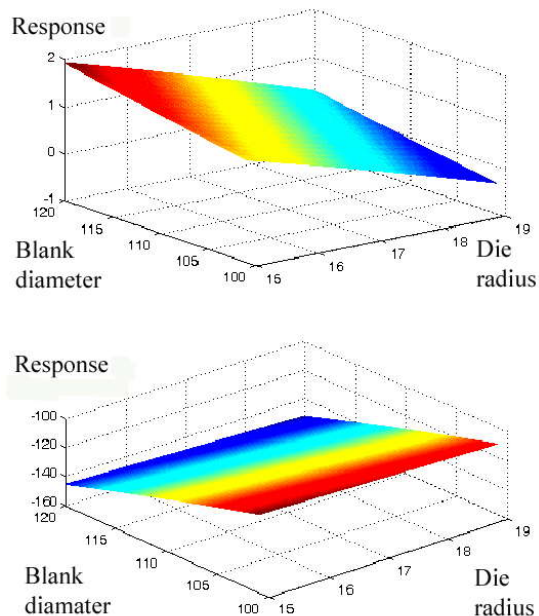
- for the direction of 90 degree:

$$y_2 = 0.8805 + x_1 \cdot 0.0050 - x_2 \cdot 0.0116 - x_1 \cdot x_2 \cdot 0.0178 - x_1^2 \cdot 0.0136 + x_2^2 \cdot 0.0080 \tag{8}$$

Figure 4 presents the forms of the surface responses in the two cases.

It can be noticed, that the interval of the values taken by each other variation is different.

The optimum values of the entrance variables have to be found (the radius of the die and the diameter of the blank) for the taken values of the 2 functions, which also must be minimized as far as possible.



**Fig 4.** The obtained response surfaces: top-for 0 degree; bottom-for 90 degree

Therefore, it was chosen to intersect the 2 surfaces shifting the first over the second one, by

measuring the height of each graphic element and intersecting them directly.

The result of this intersection is presented in figure 5.

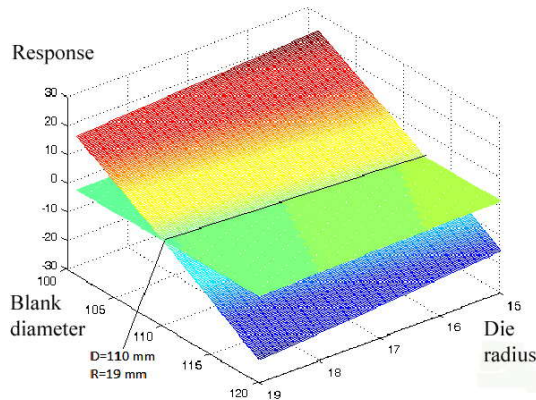


Fig 5. The intersection of the two response surfaces

The optimal combination of the parameters which assures the optimal response in the deep drawing operations is: blank diameter 110 mm and the die radius 19 mm.

## 5. Conclusions

The response surfaces method was applied for the determination of the optimum condition of deformation in deep drawing with combined restraint. The variable parameters were the die radius and the blank diameter. The response was the thickness variation and an objective function was built. The data used in response surfaces were obtained by simulation. Using simulation the thickness distribution for a number of cases was relieved, considering different deep drawing ratios and die

radii. Based on the simulation results, two quadratic models for thickness variation for the directions of 0 and 90 degree were built. Then using the design of experiments, the models coefficients were determined. An intersection of the two model equations was proposed for determining the optimum value of the parameters. It was concluded that using the response surfaces method an optimum value for the process parameters could be obtained.

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## Metoda suprafețelor de răspuns aplicată la ambutisare cu reținere combinată

### Rezumat

Ambutisarea cu reținere combinată asigură un grad de deformare mai ridicat în comparație cu metoda convențională de deformare. În timpul procesului de deformare, se manifestă o variație importantă a grosimii materialului. În lucrare, este folosită metoda suprafețelor de răspuns pentru minimizarea acestei variații. Metoda suprafețelor de răspuns consideră relația dintre parametrii procesului, în cazul studiat raza matriței și diametrul semifabricatului, și răspunsurile corespunzătoare, ca suprafețe în spațiul dimensional al variabilelor, în acest caz, variația grosimii. În final, se obține o valoare optimă a parametrilor de deformare.