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THE PROFILING OF DISK TOOL FOR GENERATION OF DISCREETLY KNOWN HELICAL SURFACES

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ABSTRACT

Most of the existing methods for profile calculation of cutting tools that work by wrapping are based on the envelope theory. For instance, methodologies for the determination of the peripheral primary tool surfaces of tools such as disk, front mill and ring tools designed to generate helical cylindrical surfaces with constant pitch are very well established for the case when an analytical description of the surfaces to be generated is available (Olivier, Gohman).
However, analytical representations of the surfaces to be generated are not always available. For instance, sometimes only a 3D discrete representation of the surface obtained from a three-dimensional numerical measuring machine or a faceted representation from CAD packages is available. In this paper, we propose a solution for the case when the surface to be generated is known only approximately at discrete points. Bezier polynomials are used to elaborate a specific methodology for profiling tools bounded by primary surfaces of revolution, which generate in the relative motion between the tool and the blank a helical surface. The results we have obtained suggest that the tool profile errors are small enough to be used in engineering applications.

KEYWORDS: disk tool, helical surfaces, tool's profiling.

1. INTRODUCTION

The design of cutting tools bounded by surfaces of revolution for the machining of helical surfaces flutes represent a problem which can be solved analytically ([2] ,[6], [7]) or numerical [8] using the enwrapping surfaces generation principles. The analytical methods based on the fundamental theorems of surfaces enveloping lead to rigorous determination of tool profiles such is the case for disk or cylindrical tools.

In many practical applications the surfaces generated by enwrapping are not associated with a helical surfaces of teeth used for torque transmission for which a very precise profile is needed. Consequently, in certain cases such as helical drills or cylindrical milling cutters absolute rigorous flute generation is not always necessary ([3], [4], [5]). Approximate methods for the realization of a methodology for profiling using a discrete representation of the surface to be generated via Bezier polynomials [1], leads in these cases to acceptable error levels.

In this paper, we develop specific algorithms for the profiling of disk-tools, reciprocally enveloping with helical flutes with constant pitch for the case when the surface generatrix is not known analytically but only by coordinates at a few points (as little as 3 or 4). Comparisons with numerical results obtained by one of the rigorous analytical methods suggest that the error level is acceptable.

2. DISK-TOOL PROFILING

In many practical situations, one may know or measure only a small number of points (as little as 4 points) along the planar generatrix of the surface to be generated. In these cases, the in-plane generatrix can be substituted by a small order (2 or 3) Bezier polynomials as illustrated in figure 1, where we have considered that the generatrix belongs to the plane perpendicular to the axis of the helical surface axis — $\vec{V}$ (Z axis):

$$X = P_X(\lambda);\ Y = P_Y(\lambda), \qquad (1)$$

where $\lambda \in [0,1]$, while $P_X(\lambda)$ and $P_Y(\lambda)$ are Bezier polynomials used to approximate the generatrix G.

Note that although in this case we consider the generatrix to be planar, the same methodology can be used when the generatrix is an arbitrary three-dimensional curve by using its projections on reference system planes.

In the helical movement of axis $\vec{V}$ and helical parameter p,

$$\begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} P_X(\lambda) \\ P_Y(\lambda) \\ 0 \end{Vmatrix} + \begin{Vmatrix} 0 \\ 0 \\ p\varphi \end{Vmatrix}, \quad (2)$$

the helical surface of constant pitch can be expressed as:

$$\Pi(\lambda,\varphi): \begin{vmatrix} X = P_X(\lambda)\cdot\cos\varphi - P_Y(\lambda)\cdot\sin\varphi; \\ Y = P_X(\lambda)\cdot\sin\varphi + P_Y(\lambda)\cdot\cos\varphi; \\ Z = p\cdot\varphi, \end{vmatrix} \quad (3)$$

where λ and φ are variables parameters.

The λ parameter is known only for a reduced number of values (3 or 4) which in many cases approximate sufficiently well via Bezier polynomials the helical surface generatrix. For second order polynomials one can express the generatrix as:

$$P\begin{vmatrix} P_X(\lambda) = \lambda^2 A_x + 2\lambda(1-\lambda)C_X + (1-\lambda)^2 B_X; \\ P_Y(\lambda) = \lambda^2 A_Y + 2\lambda(1-\lambda)C_Y + (1-\lambda)^2 B_Y. \end{vmatrix} \quad (4)$$

Similarly, for third order polynomials one can write

$$P\begin{vmatrix} P_X(\lambda) = \lambda^3 A_X + 3\lambda^2(1-\lambda)B_X + \\ +3\lambda(1-\lambda)^2 C_X + (1-\lambda)^3 D_X; \\ P_Y(\lambda) = \lambda^3 A_Y + 3\lambda^2(1-\lambda)B_Y + \\ +3\lambda(1-\lambda)^2 C_Y + (1-\lambda)^3 D_Y. \end{vmatrix} \quad (5)$$

The coefficients $A_X, A_Y, B_X, B_Y,$ C_X, C_Y, D_X, D_Y can be determined, in general, using a fitting method such as least squares from the reduced number of the actual points on the curve for which the coordinates are known. In the cases below we consider only a very small number of points such that direct determination of the coefficients is possible.

From equations (3), (4) and (5), one can determine the approximate helical surface to be generated. While an approximation, this representation can be treated as an analytical set of equations. Consequently, it is possible to use the fundamental theorems of surfaces enwrapping to determine the peripheral surface of the tool which would generate by enwrapping the desired helical surface. The helical surface can be expressed generically as:

$$\Pi(\lambda,\varphi)\begin{vmatrix} X = \Pi_X(\lambda,\varphi); \\ Y = \Pi_Y(\lambda,\varphi); \\ Z = p\cdot\varphi, \end{vmatrix} \quad (6)$$

where from (3) $\Pi_X(\lambda,\varphi)$; $\Pi_Y(\lambda,\varphi)$ are

$$\begin{aligned} \Pi_X(\lambda,\varphi) &= P_X(\lambda)\cdot\cos\varphi - P_Y(\lambda)\cdot\sin\varphi; \\ \Pi_Y(\lambda,\varphi) &= P_X(\lambda)\cdot\sin\varphi + P_Y(\lambda)\cdot\cos\varphi. \end{aligned} \quad (7)$$

The normal to the approximated helical surface $\Pi(\lambda,\varphi)$, can be written as

$$\vec{N}_\Pi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dot{\Pi}_{X\varphi} & \dot{\Pi}_{Y\varphi} & p \\ \dot{\Pi}_{X\lambda} & \dot{\Pi}_{Y\lambda} & 0 \end{vmatrix}. \quad (8)$$

Where $\dot{\Pi}$ denotes derivative with respect to either of the independent parameters. In vectorial form, the normal can be written as

$$\vec{N}_\Pi = N_X\vec{i} + N_Y\vec{j} + N_Z\vec{k}. \quad (9)$$

The Nikolaev [6] enwrapping condition can be written as

$$\left(\overrightarrow{N_\Pi}, \vec{r_1}, \vec{A}\right) = 0 \quad (10)$$

where

$$\vec{r_1} = \left[\Pi_X(\lambda,\varphi) - a\right]\cdot\vec{i} + \Pi_Y(\lambda,\varphi)\cdot\vec{j} + p\varphi\cdot\vec{k} \quad (11)$$

$$\vec{A} = -\sin(\alpha)\vec{j} + \cos(\alpha)\vec{k} \quad (12)$$

and a and α are technological parameters as shown in Fig. 1.

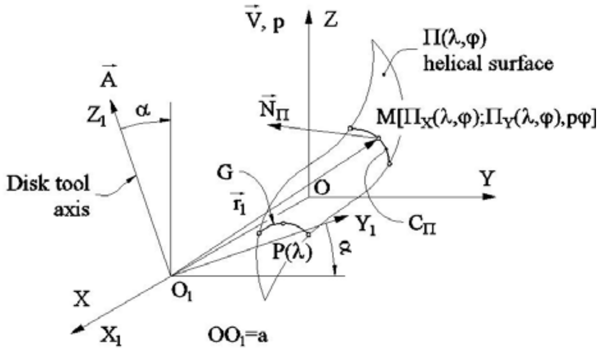

Fig. 1. The Nikolaev enwrapping condition

The enwrapping condition (10) can be written in approximated form as

$$-\varepsilon \le \begin{vmatrix} N_X & N_Y & N_Z \\ \Pi_X(\lambda,\varphi) - a & \Pi_Y(\lambda,\varphi) & \Pi_Z(\lambda,\varphi) \\ 0 & -\sin\alpha & \cos\alpha \end{vmatrix} \le \varepsilon \quad (13)$$

where ε is sufficiently small positive number.

From (13) one can determine points belonging to the characteristic curve C_Π illustrated in figure 1. This would be accomplished for 3 or 4 values of the λ parameter, corresponding to the approximation polynomial level for the helical surface generatrix (4) or (5). For a second order polynomials the characteristic curve can be expressed as

$$C_\Pi = \begin{Vmatrix} X_{C_\Pi,\lambda=0} & Y_{C_\Pi,\lambda=0} & Z_{C_\Pi,\lambda=0} \\ X_{C_\Pi,\lambda=1/2} & Y_{C_\Pi,\lambda=1/2} & Z_{C_\Pi,\lambda=1/2} \\ X_{C_\Pi,\lambda=1} & Y_{C_\Pi,\lambda=1} & Z_{C_\Pi,\lambda=1} \end{Vmatrix}. \quad (14)$$

By expressing these coordinates in the tool's reference system illustrated in figure 1, one determines these 3 or 4 points on the disk tool S,

$$\begin{Vmatrix} X_{1_{C_\Pi}} \\ Y_{1_{C_\Pi}} \\ Z_{1_{C_\Pi}} \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{Vmatrix} \cdot \left[\begin{Vmatrix} X_{C_\Pi} \\ Y_{C_\Pi} \\ Z_{C_\Pi} \end{Vmatrix} - \begin{Vmatrix} a \\ 0 \\ 0 \end{Vmatrix}\right]. \quad (15)$$

The disk tool's characteristic curve is shown in Fig. 2 and can be written as a set of coordinates,

$$\left[X_{1_{C\Pi}}, Y_{1_{C\Pi}}, Z_{1_{C\Pi}}\right], \quad (i = 1,2,3). \qquad (16)$$

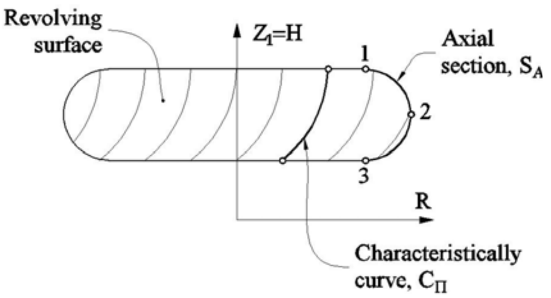

Fig. 2. Disk tool's peripheral surface axial section

To determine S_A, the axial profile of the tool, one observes that:

$$S_A \left| \begin{array}{l} \left(Z_{1_{C\Pi}}\right)_i = H_i; \\ \left(\sqrt{X^2_{1_{C\Pi}} + Y^2_{1_{C\Pi}}}\right)_i = R_i, \end{array} \right. \quad (i = 1,2,3), \qquad (17)$$

for a second order polynomial, or *i=1,2,3,4* for a third order polynomial.

The process is thus completed: starting only from three or four points on the planar generatrix of the helical surface one can obtain an approximate profile of the required tool. This requires fewer computations than in the case of classical methods. Moreover, it allows for the determination of the tool profile even when the desired profile to be obtained is known only at very reduced number of points, such as obtained from measuring devices such as OMMs (on machine measuring).

Finally, the results below suggest that the intrinsic approximation of the method is quite acceptable as the comparisons with results obtained using classical methods are very satisfactory.

3. APPLICATIONS

3.1. Helical surface with constant pitch with rectilinear generatrix profile

We first assess the accuracy of the new method in the case of a disk tool used to manufacture a worm with straight line generatrix.

In Fig. 3, are represented the reference system, the rectilinear generatrix and the characteristic points in this case:

- *XYZ* is the reference system regarding which is defined the helical surface's generatrix, Δ;
- characteristic points along the generatrix, $A\left[X_A, Y_A, Z_A\right]$ and $B\left[X_B, Y_B, Z_B\right]$;
- κ — constant angular parameter;
- d_0 — the diameter of cylinder coaxially.

The helical surface to be generated is also shown.

The substitution polynomial is a first degree polynomial and thus the generatrix equations are:

$$\Delta \left| \begin{array}{l} X = \dfrac{d_0}{2}; \\ Y(\lambda) = \lambda A_Y + (1-\lambda) B_Y; \\ Z(\lambda) = \lambda A_Z + (1-\lambda) B_Z. \end{array} \right. \qquad (18)$$

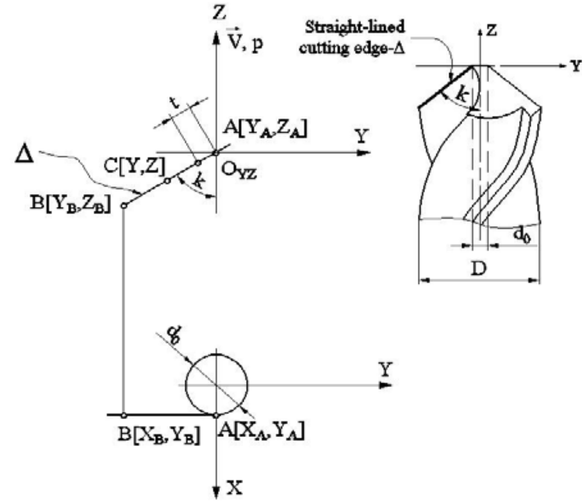

Fig. 3. Reference system. Helical surface's generatrix

From the transformation(3) one obtains the approximated helical surface:

$$\Pi_{(\lambda,\varphi)} \left| \begin{array}{l} X = \dfrac{d_0}{2} \cdot \cos(\varphi) - Y(\lambda)\sin(\varphi); \\ Y = \dfrac{d_0}{2} \cdot \sin(\varphi) + Y(\lambda)\cos(\varphi); \\ Z = Z(\lambda) + p \cdot \varphi. \end{array} \right. \qquad (19)$$

The normal to the approximated helical surface can be obtained using (8) and (9) as:

$$\begin{array}{l} N_{\Pi_X} = p\left[A_Y - B_Y\right]\cos(\varphi) - \\ -\left[A_Z - B_Z\right] \cdot \left[\dfrac{d_0}{2}\cos(\varphi) - Y(\lambda) \cdot \sin(\varphi)\right]; \\ N_{\Pi_Y} = +p\left[A_Y - B_Y\right]\sin(\varphi) - \\ -\left[A_Z - B_Z\right] \cdot \left[\dfrac{d_0}{2}\sin(\varphi) + Y(\lambda) \cdot \cos(\varphi)\right]; \\ N_{\Pi_Z} = -\left[\dfrac{d_0}{2}\cos(\varphi) - Y(\lambda) \cdot \sin(\varphi)\right] \cdot \left(A_Y - B_Y\right) \cdot \sin(\varphi) + \\ +\left[\dfrac{d_0}{2}\sin(\varphi) + Y(\lambda) \cdot \cos(\varphi)\right] \cdot \left(A_Y - B_Y\right) \cdot \cos(\varphi). \end{array} \qquad (20)$$

Following the methodology one obtains

$$\begin{array}{l} \vec{r}_1 = \left[\dfrac{d_0}{2}\cos(\varphi) - Y(\lambda) \cdot \sin(\varphi) - a\right] \cdot \vec{i} + \\ +\left[\dfrac{d_0}{2}\sin(\varphi) + Y(\lambda) \cdot \cos(\varphi)\right] \cdot \vec{j} + \\ +\left[Z(\lambda) + p \cdot \varphi\right] \cdot \vec{k}, \end{array} \qquad (21)$$

and the enwrapping condition (13) together with (19), determine the characteristic curve on the approximated helical surface.

The coefficients of the first order polynomial can then be determined as shown in table 1.

Table 1. The 1st order ploynomial

t	Primary profile	λ	Approximate polynomial coefficient
t_A	$t_A = 0$ $Y = Y_A$ $Z = Z_A$	0	$B_Y = Y_A$ $B_Z = Z_A$
t_B	$t_B = \left(\left[Y_A - Y_B \right]^2 + \right.$ $\left. + \left[Z_A - Z_B \right]^2 \right)^{1/2}$	1	$A_Y = Y_B$ $A_Z = Z_B$

3.2. Disk tool for helical drill's flute generation

The disk tool for the profiling of the helical drill flute shown in figure 3 can be determined. Consider the following data: external drill's diameter, *D=20* mm; drill's core diameter, $d_0=0.16 \cdot D=3.2$ mm; distance between the tool axis and the drill axis, *a=50* mm; inclination angle of helix on the external drill's diameter, $\omega = \pi / 6$; helical parameter of drill's flute, $p = \dfrac{D}{2 \cdot tg\omega}$.

Coordinates of the four points (in YZ system) belonging to generatrix, are: A=[0; 0], B=[-4.23; -1.54], C=[-7.03; -3.18], D=[-10.0; -4.93].

In table 2, we present the axial section of the disk tool calculated based on an analytical method of surface reciprocally enwrapping and by approximation with the Bezier polynomials.

Table 2.

λ	Approximated profile		Theoretical profile		Error [mm]
	R [mm]	H [mm]	R [mm]	H [mm]	
0.00	**48.400**	**0.000**	**48.400**	**0.000**	**0.000**
	48.338	1.053	48.343	1.058	0.007
	48.151	2.098	48.155	2.103	0.006
	47.840	3.126	47.841	3.126	0.001
0.33	**47.710**	**3.459**	**47.709**	**3.461**	**0.002**
	47.404	4.127	47.401	4.127	0.003
	46.843	5.093	46.838	5.094	0.006
	46.157	6.016	46.154	6.014	0.003
0.66	**45.634**	**6.595**	**45.630**	**6.599**	**0.005**
	45.342	6.884	45.339	6.888	0.005
	44.395	7.685	44.403	7.686	0.008
	43.306	8.405	43.311	8.409	0.007
1.00	**42.053**	**9.020**	**42.053**	**9.020**	**0.000**

The error is defined as the distance measured along the normal of the approximated profile to the theoretical profile regarded as reference.

In Fig. 4, the axial section of the disk-tool primary peripheral surface is shown.

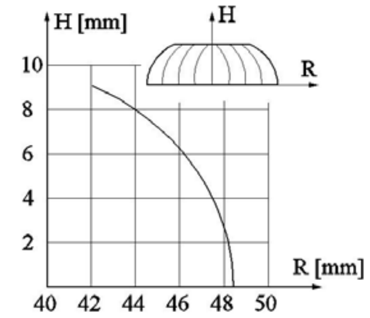

Fig. 4. The axial section of disk-tool primary peripheral surface

The error between the two axial sections of disk-tool is very small, so the method although approximate, is still quite accurate for engineering practice.

3.3. Disk-tool to generate a worm with circular generatrix in the cross plane

In Fig. 5, is presented the crosing section of helical surface with constant pitch and circular generatrix.

Consider that the following coordinates are known:

- the center $O_C[X_{OC}, Y_{OC}]$;

- coordinates of the circular arc ends, $A\ [X_A, Y_A]$ and D $[X_D, Y_D]$.

The analytical equations of the circular profile:

$$X = X_{O_C} + R\cos\theta; \ Y = Y_{O_C} - R\sin\theta. \quad (22)$$

The Bezier substitution polynomials of second or third order are shown in table 3. Note that the *Z* coordinates of all defined points on the circular arc, may be considered constant *Z=H*, typically *H=0*.

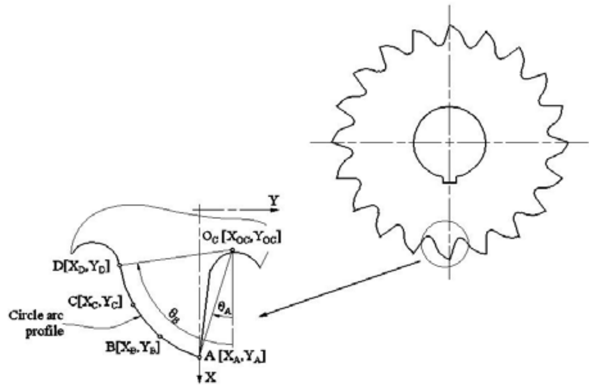

Fig. 5. Circular profile in frontal plane (disk-mill tool with helical teeth)

Table. 3. The 3rd order polynomial

θ	**Primary profile**	λ	**Approximate polynomial coefficient**
θ_A	$X_A = X_{O_C} + R\cos\theta_A$ $Y_A = Y_{O_C} - R\sin\theta_A$	0	$D_X = X_A$ $D_Y = Y_A$
θ_B	$\theta_B = \theta_A + \frac{\theta_D - \theta_A}{3}$ $X_B = X_{O_C} + R\cos\theta_B$ $Y_B = Y_{O_C} - R\sin\theta_B$	$\frac{1}{3}$	$B_X = \frac{18\cdot X_C - 9\cdot X_B}{6} + \frac{2\cdot X_A - 5\cdot X_D}{6}$ $B_Y = \frac{18\cdot Y_C - 9\cdot Y_B}{6} + \frac{2\cdot Y_A - 5\cdot Y_D}{6}$
θ_C	$\theta_C = \theta_A + \frac{2(\theta_D - \theta_A)}{3}$ $X_C = X_{O_C} + R\cos\theta_C$ $Y_C = Y_{O_C} - R\sin\theta_C$	$\frac{2}{3}$	$C_X = \frac{-5\cdot X_A + 2\cdot X_D}{6} + \frac{18\cdot X_B - 9\cdot X_C}{6}$ $C_Y = \frac{-5\cdot Y_A + 2\cdot Y_D}{6} + \frac{18\cdot Y_B - 9\cdot Y_C}{6}$
θ_D	$X_D = X_{O_C} + R\cos\theta_D$ $Y_D = Y_{O_C} + R\sin\theta_D$	1	$A_X = X_D$ $A_Y = Y_D$

In Fig. 6 and table 4, are presented the numerical coordinates of the axial profile of disk-tool, reciprocally enveloping with surfaces with circular frontal generatrix, determined by the two methods with the error outlined in the last column, where (see figure 7):

$$p = 75mm; R = 55mm; O_c : [125; 0.0];$$
$$\theta_A = 0; \theta_D = 2\pi/8; a = 280mm,$$

(where *a* is the distance between piece origin and disk tool origin, see Fig. 1).

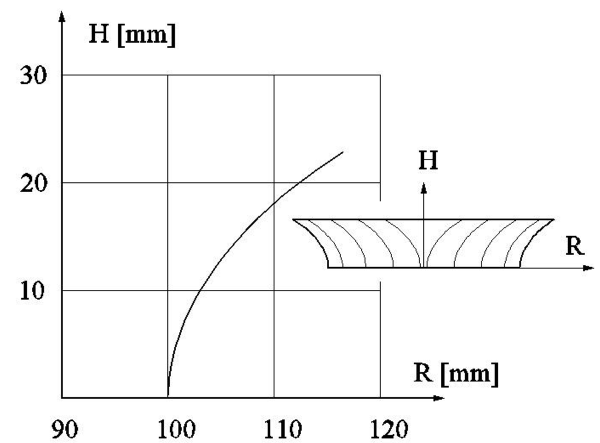

Fig. 6 Axial profiles of disk-tool

Table 4.

λ	**Approximated profile**		**Theoretical profile**		**Error [mm]**
	R [mm]	**H [mm]**	**R [mm]**	**H [mm]**	
0.00	**100.004**	**0.347**	**100.004**	**0.338**	**0.010**
0.10	100.303	3.227	100.316	3.237	0.016
0.20	101.060	6.023	101.063	6.017	0.007
0.33	**102.668**	**9.522**	**102.658**	**9.513**	**0.013**
0.40	103.691	11.173	103.683	11.172	0.008
0.50	105.426	13.493	105.422	13.495	0.004
0.60	107.364	15.648	107.364	15.647	0.001
0.66	**108.733**	**16.985**	**108.726**	**16.972**	**0.015**
0.80	111.687	19.506	111.701	19.505	0.015
0.90	114.013	21.236	114.018	21.228	0.009
1.00	**116.422**	**22.847**	**116.420**	**22.845**	**0.004**

Note: When the points are mesured on the helical surface's generatrix, be A[X_A, Z_A]; B[X_B, Z_B]; C[X_C, Z_C]; D[X_D, Z_D] these points, we defined the λ parameter values by:

$$\lambda_B = \frac{\left|\overline{AB}\right|}{\left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right|};$$
$$\lambda_C = \frac{\left|\overline{AB}\right| + \left|\overline{BC}\right|}{\left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right|}, \quad (23)$$

where $\left|\overline{AB}\right|, \left|\overline{BC}\right|, \left|\overline{CD}\right|$ are the *AB, BC* and *CD* strainght line segment modulus.

4. SOFTWARE CONSIDERATIONS

A software application developed in Java programming language was specially designed for this purpose. In Fig. 7, we present a screenshoot of the java applet, where we can define:

- coordinates of an arbitrary number of points, mesured on generatrix curve (numerical values can be imported from CSV files);
- outer diameter of the helical surface (D);
- the tool type (disk tool) inour case).

To estimate the error for the method proposed in this paper, and for comparative purpose, we can also use in our application a series of analytically defined generatrix curves (circle arc, straight line, involute and non-analytically profiles, defined by measured points).

Also, the applet can represent planar projection of generatrix curve, axial tool profile (bottom-right corner), a 3D representation of the helical surfaces and normal vectors (top-left corner).

Numerical results can be exported to comma separated values files.

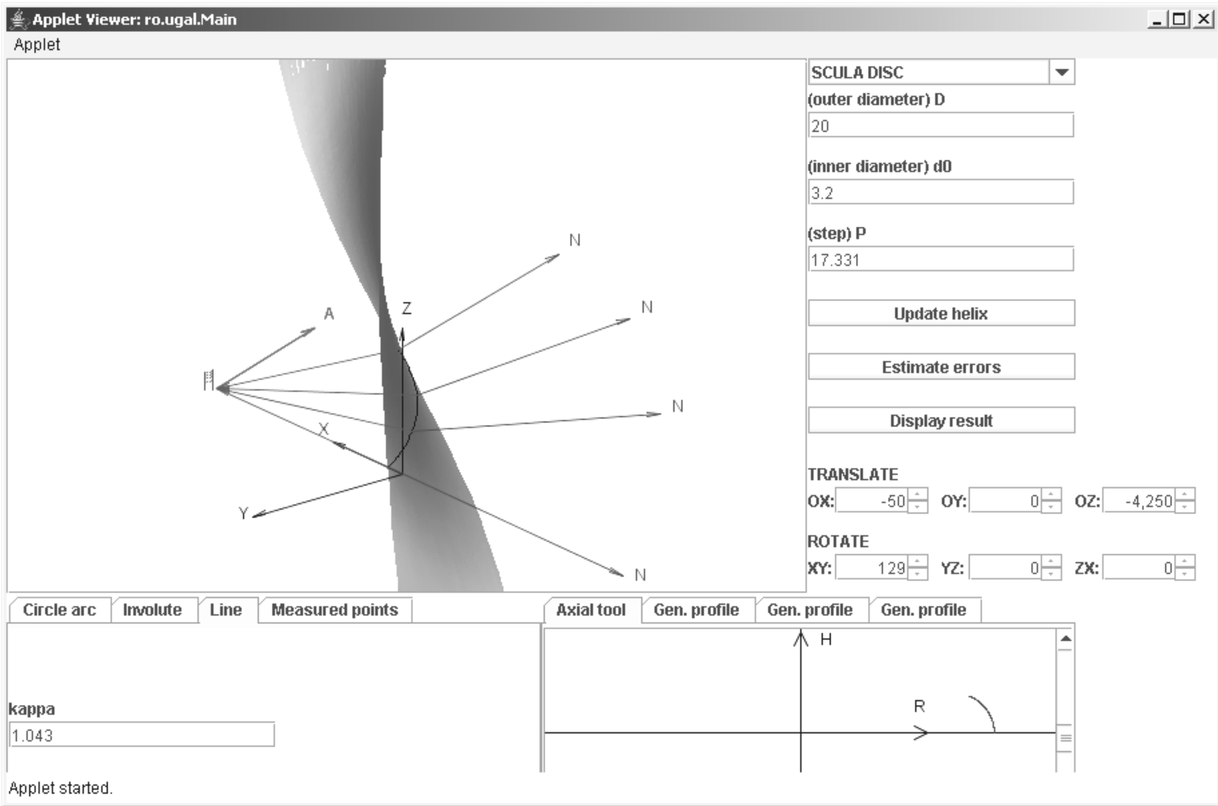

Fig. 7. Applet screenshot (disk tool)

5. CONCLUSIONS

In this paper we have presented a novel method to approximate via Bezier polynomials of small order a cylindrical helical surface with constant pitch and in-plane generatrix known in discreet form by a reduced number of points (3 or 4).

Specific algorithms were developed for the determination of the axial profile of the associated disk-tool, reciprocally enveloping with the helical surface, known in discreet form and expressed by Bezier replacing polynomials.

Two examples illustrated the accuracy of the method. Besides the computational advantage (faster execution), the main appeal of the method is that the profile to be generated can be represented by the coordinates of a small number of points. These points can be eventually obtained from physical measurements on the generatrix.

The presented algorithm is characterized by a satisfactory precision for typical industrial practice, especially for surfaces which are not associated with large load torque such as gears.

The proposed algorithm is based on the Nikolaev theorem, but may be applied to other method known in the reciprocally enveloping surfaces studies.

The proposed algorithm and also the software, are faster and easier to apply than traditional methods.

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