THE ANNALS OF "DUNĂREA DE JOS" UNIVERSITY OF GALAȚI
FASCICLE V, TECHNOLOGIES IN MACHINE BUILDING,
ISSN 1221- 4566, 2009

A NEW APPROACH OF THE CUTTING PROCESS DYNAMICS

Gabriel Frumuşanu, Alexandru Epureanu

Dunărea de Jos University of Galați, Romania
email: gabriel.frumusanu@ugal.ro

ABSTRACT

It is already more and more accepted by the researchers working in the manufacturing area, that the cutting process dynamics is nonlinear and, more than that, there is proof to support this assessment, under certain conditions, of chaos. The paper presents the authors' research, performed in order to reveal the potential chaotic character of the cutting process dynamics. A cutting process characteristic parameter – the cutting force - was measured and its variation was analyzed by using Chaos Theory specific tools. Experimental results and conclusions are also presented.

KEYWORDS: cutting dynamics, chaos, cutting force, time series.

1. INTRODUCTION

Classical theory of cutting stability starts from the regenerative phenomenon, according to which the current cutting cycle is determined by the perturbations appeared during previous cycle and its fundamental physical model is shown in Fig.1. Elastic deformation from previous cycle, *y(t-T)*, induces cutting force *F* variation, which further determines *y(t)* deformation of mechanical structure during current cycle. As consequence, chip real thickness is given by

$$BD = a(t) = a_0(t) - y(t) + y(t-T), \qquad (1)$$

where a_0 means the nominal chip thickness.

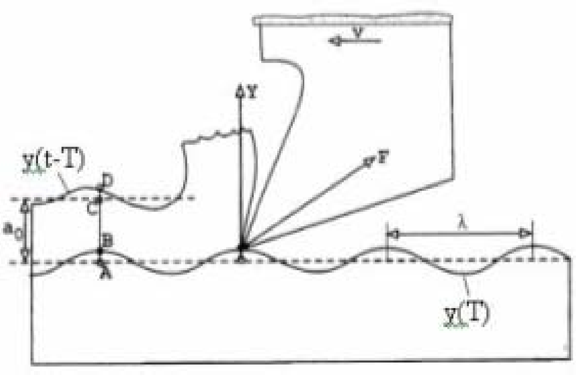

Fig.1 Cutting process physical model

The system diagram describing perturbations regeneration phenomenon is shown in Fig.2 and enables to find system transfer function,

$$Y = \frac{a(s)}{a_0(s)} = \frac{1}{1 + Y_1(s)Y_2(s)\left(1 - e^{-sT}\right)}. \qquad (2)$$

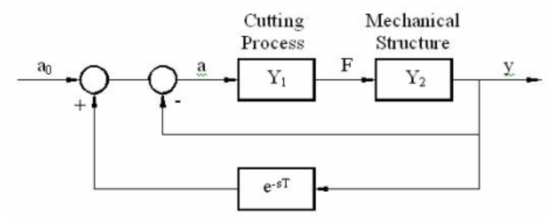

Fig.2 Manufacturing system diagram:
a_0/a – nominal/real chip thickness; y – system elastic deformation; T – cutting cycle duration; F – cutting force.

Stability is approached based on the general stability criterion, according to which if one of the characteristic equation solutions has a positive real part, then the system is unstable. Referring to cutting process, the characteristic equation is

$$1 + Y_1(s)Y_2(s)\left(1 - e^{-sT}\right) = 0 . \qquad (3)$$

To find the limit separating stable from unstable domains, we must impose to the characteristic equation pure imaginary solutions, resulting in the following equation of stability limit:

$$1+Y_1(j\omega)Y_2(j\omega)\left(1-e^{-j\omega T}\right)=0\,. \qquad (4)$$

In the current cutting stability theory, transfer function $Y_1(j\omega)$ is considered to be a real constant, by entirely neglecting cutting process dynamics. On the other hand, the system's mechanical structure is considered linear, which allows to look at $Y_2(j\omega)$ as the frequency characteristic of the mechanical structure (that can be experimentally found). Under these conditions, stability limit results from the relation

$$A(j\omega)=B(j\omega)\,, \qquad (5)$$

where

$$A(j\omega)=Y_1(j\omega)Y_2(j\omega) \qquad (6)$$

and

$$B(j\omega)=\frac{1}{e^{-j\omega T}-1}\,. \qquad (7)$$

To solve this equation, a graphic-analytical method is used, Fig.3, where, by increasing $A(j\omega)$ constant, the expanding of the frequency characteristic $Y_2(j\omega)$ is obtained, until touching $B(j\omega)$ line, when instability phenomenon appears.

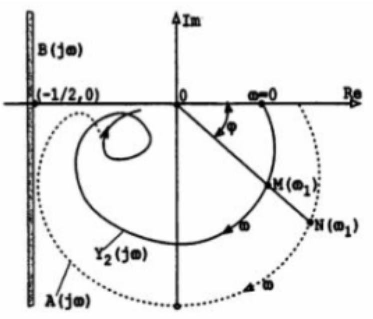

Fig.3 Graphic-analytical method to find the stability limit [2]

Recent evolutions in the cutting stability theory are concretizing through a significant number of papers, dedicated to theoretically investigate new methods and models to study manufacturing processes dynamics, [4-10], by going away from the classical cutting model and based on new approaches. It should be mentioned that, in almost all the cases, a non-linear model is firstly developed and then, a comparison to the real cutting processes is done by numerical modelling.

2. LIMITS OF CURRENT THEORY CONCERNING THE STABILITY

Even by considering the recent scientific contributions to the cutting process stability theory, a critical analysis reveals the following limits and unclarified matters:

- Cutting process dynamics is not considered, although in the cutting instability phenomenon it plays an essential role. Thus, it cannot be explained the dependence between cut material – chip shape – process stability (e.g. comparative stability between steel and bronze).
- There is no explanation for dependence between cutting speed and feed rate, on one hand and stability limit, on the other hand, as this dependence can be experimentally observed.
- Current approach cannot explain neither why instability only appears when wave length of traces let by self-excited vibration on workpiece's surface has values between 0.5 and 12 mm and nor why at the middle of the interval the stability has a minimum level, as the authors of this paper observed during their experimental research.
- Current theory cannot enable to find functioning point position referred to stability limit. More precisely, it cannot appreciate the reserve of stability existing at a given moment.
- To find the stability limit, in the context of the actual stability theory, means to know the system's frequency characteristic. To obtain it, supposes to follow a complex experimental plan. But, right in the moment when the tool moves along the worked piece generating line, the frequency characteristic permanently changes. Thus, the current theory does not offer *the possibility of monitoring, in real time, the technological system's reserve of stability*. This is the reason to intervene on the system only after it reached the unstable functioning domain.

3. THE POTENTIAL CHAOTIC CHARACTER OF THE CUTTING PROCESS DYNAMICS

As it is known, cutting processes stability classical theory is based on a linear dynamics approach, developed by starting from the block-scheme shown in Fig.2, where the transfer function Y_1 (characterizing the cutting process) is a constant while the transfer function Y_2 (characterizing the technological system) is a linear function. This approach cannot explain the stability limit dependence to machining speed and feed. On the other hand, there are situations when the instability phenomenon cannot be understood by using the classical approach.

Research already developed by the authors of this paper lead to the idea that *cutting should be treated as chaotic process, with bifurcation points, attractors and limit cycles* – the dynamics' transition from stable to unstable or inverse meaning, in fact, the transition between two limit cycles. This could further lead to a better understanding of the conditions to pass from stability to instability and also to the opportunity of developing an intelligent tool for realizing the stability control.

3.1. The largest Lyapunov exponent of cutting force time series

The main feature that characterizes the chaotic character of a certain process is “the largest Lyapunov exponent”, which may be defined as

$$\lambda_1(i) = \frac{1}{i \cdot \Delta t} \cdot \frac{1}{M-i} \sum_{j=1}^{M-i} ln \frac{d_j(i)}{d_j(0)}, \qquad (8)$$

where Δt is time series sampling period, $d_j(i)$ – the distance between the j^{th} pair of nearest neighbors after i discrete time-steps, M – the number of reconstructed points. A positive largest Lyapunov exponent is sufficient to diagnose chaos and represents local instability in a particular direction.

By assuming that there is a connection between cutting force variation and cutting process dynamics character, we calculated this exponent in the case of cutting force time-series. To do this, first, cutting force values must be discretely measured for a certain time interval, by a Δt increment and recorded as a time series. We used a SPIDER 8 data acquisition device, with 9600 scan/s; it was placed on the cutting tool, as close as possible to the cutting edge, in order to minimize other devices’ inertial forces influence. Then, the file including time series record can be analyzed by using a dedicated soft [3], to calculate the largest Lyapunov exponent.

The above mentioned algorithm was applied to analyze the turning process. Cutting tests were performed on a turning machine, by manufacturing exterior cylindrical surfaces (40 mm diameter) of ordinary steel workpieces, with a cutting tool having a setting angle of 90º. Different rotation speeds (between 100 and 800 rot/min) were successively used; the feed was of 0.2 mm/rot while the depth of cut was set to 6 mm. The cutting force was discretely sampled by a time increment Δt = 1/9600 s.

We have chosen only eight of the files containing cutting force values, considered representatives (because some of the files are similar), and we have analyzed them by using the special dedicated soft. The values obtained for largest Lyapunov exponent, λ, in these cases were: 0.5953, 0.6450, 0.6565, 0.3210, 0.4546, 0.4347, 0.5738, and 0.5983. It means that the considered cutting processes should be treated as chaotic.

3.2. Poincaré map of cutting force time series

In the mathematical study of dynamical systems, a map refers to a time-sampled sequence of data $\{x(t_1), x(t_2), \ldots, x(t_n), \ldots, x(t_N)\}$ with the notation $x_n = x(t_n)$. A simple deterministic map is one in which the value of x_{n+1} can be determined from the values of x_n. This is often written in the form

$$x_{n+1} = f(x_n). \qquad (9)$$

The idea of a map can be generalized to more than one variable. For example, suppose we consider the motion of a particle as displayed in the phase plane $(x(t), \dot{x}(t))$. However, if instead of looking at the motion continuously, we look only at the dynamics at discrete times, then the motion will appear as a sequence of dots in the phase plane. If $x_n \equiv x(t_n)$ and $y_n \equiv \dot{x}(t_n)$, this sequence of points in the phase plane represents a two-dimensional map:

$$\begin{vmatrix} x_{n+1} = f(x_n, y_n); \\ y_{n+1} = g(x_n, y_n). \end{vmatrix} \qquad (10)$$

When the sampling times t_n are chosen according to certain rules, this map is called a Poincaré map. When there is a driving motion of period T, a natural sampling rule for a Poincaré map is to choose $t_n = nT + \tau_0$. This allows one to distinguish between periodic motions and non-periodic motions.

We intended to confirm the chaotic character of cutting process dynamics by drawing Poincaré maps based on cutting force, F, values time series. By using a dedicated soft to calculate points’ co-ordinates and AutoCad to realize the graphical representation of dependence between $\dot{F}$ and F, Poincaré maps were obtained. If normally the picture has the shape of a single cloud of points, in some cases we found maps as it can be seen in Fig.4.

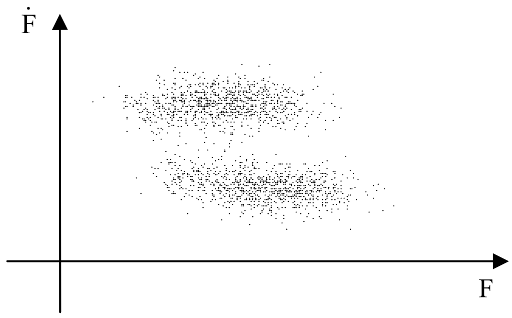

Fig.4 Poincaré maps drawn based on cutting force time series

Poincaré map from Fig. 4 was drawn by using a file including 4000 measured values of the cutting force, during a turning process performed with a rotation speed of 500 rot/min.

The existence of two sets of points suggests the existence of a bifurcation point between two limit cycles – aspect characteristic to systems with chaotic dynamics.

If we consider the Logistic model, the most popular one-dimensional chaotic model,

$$x_{n+1} = r \cdot x_n (1 - x_n), \; n \geq 1, \qquad (11)$$

where $0 < x_1 < 1$ and the control parameter, r, takes values close to 4, then one of its properties becomes interesting for us: if r is around the value of 3.83, the Logistic model's map consists in period-3 cycles, interrupted by sections with chaotic aspect (Fig.5), while in other cases (e.g. $r > 3.85$ or $r < 3.8$) the map is completely chaotic.

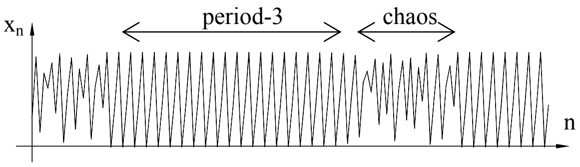

Fig.5 Logistic model map, r = 3.8282

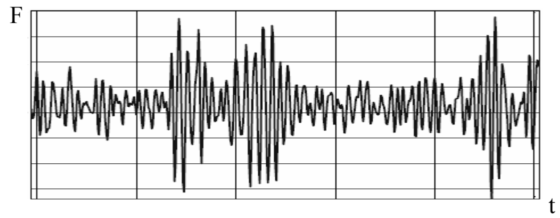

Fig.6 Cutting force variation, n = 400 rot/min

In Fig. 6, one can see that cutting force variation may be similar to the map from Fig.5 (referring to the succession between two types of sections).

If a Logistic-type model (with a certain value for r) will be found to characterize the cutting force variation during a given manufacturing process, then, by monitoring the cutting force, in fact we will be able to know, in real time, the value of the control parameter. By accepting that the commutation between chaotic and periodic sections (Fig.5) means, by analogy, when referring to the cutting process, the commutation between stable and unstable dynamics, a tool to find when the instability risk occurs could be developed starting from here.

4. CONCLUSIONS

The limits of the classic theory concerning cutting process stability and the current expectations from the modern manufacturing systems require a new approach in this domain.

Because it cannot refer the manufacturing system operating point to its stability limit, the current theory does not offer the possibility of entirely exploiting the technological system reserve of stability. The interventions in the system can be done only after it reached the unstable functioning domain. Any kind of stability prognosis is impossible, especially on-line, and that is why the actual technological systems do not have a system to control stability.

The results presented above in this paper open the way for going over the enounced limits, through a new theory concerning cutting process stability, based on chaotic models.

Grounded on the new theory, an intelligent system to control cutting stability could be imagined and realized, leading to a complete exploitation of technological system resources of productivity, by working with cutting regimes more intense, closer to the limit of stability domain. Thus, it will result both a maximisation of manufacturing processes efficiency and a superior quality of manufactured surfaces, by eliminating the risk of the instability appearance.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the Romanian Ministry of Education, Research and Innovation through Grant PN-II ID 794/2008.

REFERENCES

[1] Abarbanel, H.D.I., *Analysis of Observed Chaotical Data*, Springer, New York, 1996.
[2] Epureanu, Al., *Sisteme tehnologice. Precizie. Stabilitate. Economicitate*, Editura Fundației Universitare "Dunărea de Jos", Galați, 2004.
[3] Frumuşanu, G., *Algorithm and Program to Find Largest Lyapunov Exponent in One-dimensional Time-series Case*, Proceedings of TMCR'07 International Conference, Chişinău, Republica Moldova, 2007, vol.2, pp.338-341.
[4] Litak, G. et al., *Nonlinear Analysis of Experimental Time Series of a Straight Turning Process*, Meccanica 39, 2004, pp. 105-112.
[5] Long, X.-H., Balachandran, B., Mann, B.P., *Dynamics of milling processes with variable time delays*, Nonlinear Dynamics, DOI 10.1007/s1 1071-006-9058-4, vol.47, no.1-3/Jan 2007.
[6] Marghitu, D.B. et al., *Dynamics in ortogonal turning process*, Chaos, Solitons & Fractals 12, 2001, pp. 2343-2352.
[7] Moon, F.C., Kalmar-Nagy, T., *Nonlinear models for complex dynamics in cutting materials*, Philosophical Transactions of Royal Society London A, 359, 2001, pp. 695-711.
[8] Park, S. S., *Robust regenerative chatter stability in machine tools*, International Journal of Advanced Manufacturing Technologies, DOI 10.1007/s00170-006-0778-x, 2006.
[9] Wang, X.S. et al., *Nonlinear dynamics of regenerative cutting processes – Comparison of two models*, Chaos, Solitons & Fractals 29, 2006, pp. 1218-1228.
[10] Wiercigroch, M., *Chaotic Vibration of a Simple Model of the Machine Tool – Cutting Process System*, ASME Journal of Vibrations and Acoustics, 1997, Vol.119, pp. 468-475.