Approximation of the Gear Cutter Profile Used in the Generation of Interior Poliform Surfaces

PhD. eng. Oancea Nicolae, Mihăluță Marius,
PhD. eng. Teodor Virgil
„Dunărea de Jos" University of Galați

ABSTRACT

They are known the profiling principles for gear cutter for the mortising of hub crossing profiles with hexagonal or square section. In this paper is proposed a new approximation methodology for polyformes gear cutter profiles which have a machining technology more simple, without need a numerical controlled machine.
***Keywords**: polyform surfaces, gear cutter, profile approximation.*

1. Introduction

The pinion cutters used in the generation of interior hexagonal surfaces have profiles determined as being reciprocally enwrapping to the straight profiles of the straight line's vortex which makes up the polygonal contour of the hexagonal shaft [1] [3].

Once determined, the pinion cutter's profile is to be physically created as the pinion cutter's cutting edge.

The common method used for this is the profiling of the pinion cutter on the grinding machine, the tool's flank being made at the same time.

This is a secure method, but a bit complicated, taking into account the fact that only for the plane flanks the pinion cutter's form is identical to that commanded on the grinding machine.

In this paper, an approximating solution of the pinion cutter's profile, generator of the hexagonal bushing using a polyform [2] grinded shaft's cross section, is suggested.

This solution has the advantage of simplifying the pinion cutter's fabrication technology. Obviously, a very rigorous tool needs a grinding process for the tool's flank but, for some practical needs, the suggested approximation can work very well.

2. The Profiling of the Pinion Cutter for the Cutting of Hexagonal Bushings

In figure 1, the reference systems, the centrode associated to the rolling profiles and the interior polygonal profile obtained after the cutting process are presented for a non-involute profile.

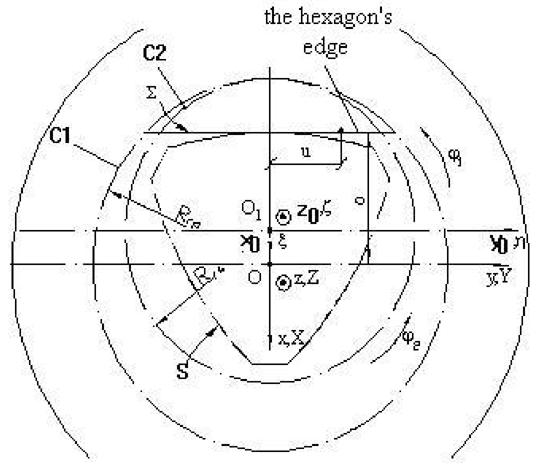


***Fig. 1.** Reference systems, rolling centrods, enwrapping profiles*

In this case, the fact that the blank's rolling radius coincides with the radius of the hexagon's circumscribed circle can be accepted.

Related to figure 1, the reference systems are defined:

- xyz, the global reference system which has the origin over posed to the blank's rotation axis;
- XYZ – the relative system associated to the blank;
- $\xi\eta\zeta$ – the mobile system associated to the pinion cutter.

The transmission ratio is defined using the rolling condition without sliding of the two axodes, C1 and C2, of R_{rp} and R_{rs} radii:

$$i = \frac{\varphi_2}{\varphi_1} = \frac{R_{rp}}{R_{rs}}, \quad (1)$$

where:

- φ_1 is the blank's rotation angle, in a rotation movement by constant angular speed around the Z axis;
- φ_2 – the tool's rotation angle, in a rotation movement by constant angular speed around the ζ axis;

The hexagonal shaft's flank's surface is described by the following equations:

$$\Sigma \begin{vmatrix} X = -a; \\ Y = u; \\ Z = t, \end{vmatrix} \quad (2)$$

with u and t variable.

The distance between the blank and the tool axis, measured along the x axis, is:

$$A_{12} = R_{rp} - R_{rs}.$$

The coordinate transformation between the fixed reference systems being known as:

$$x_0 = x - A,$$

with:

$$A = \begin{Vmatrix} -A_{12} \\ 0 \\ 0 \end{Vmatrix},$$

the relative movements between the mobile reference systems, $\xi\eta\zeta$ and XYZ, are determined:

$$\xi = \omega_3(-\varphi_2)\left[\omega_3^T(\varphi_1)X - A\right]. \quad (3)$$

Thus, according to the (3) movement, the surfaces family, (4), is determined in the tool's referrence system:

$$(\Sigma)_{\varphi_1} = \begin{vmatrix} \xi = -a\cos(\varphi_1-\varphi_2) - u\sin(\varphi_1-\varphi_2) + A_{12}\cos\varphi_2; \\ \eta = -a\sin(\varphi_1-\varphi_2) + u\cos(\varphi_1-\varphi_2) - A_{12}\cos\varphi_2; \\ \zeta = t. \end{vmatrix} \quad (4)$$

The envelope of the $(\Sigma)_{\varphi 1}$ family give the pinion cutter profile.

The enwrapping condition is:

$$\varphi_1 = \arcsin\left[\frac{i-1}{iA_{12}}u\right], \quad (5)$$

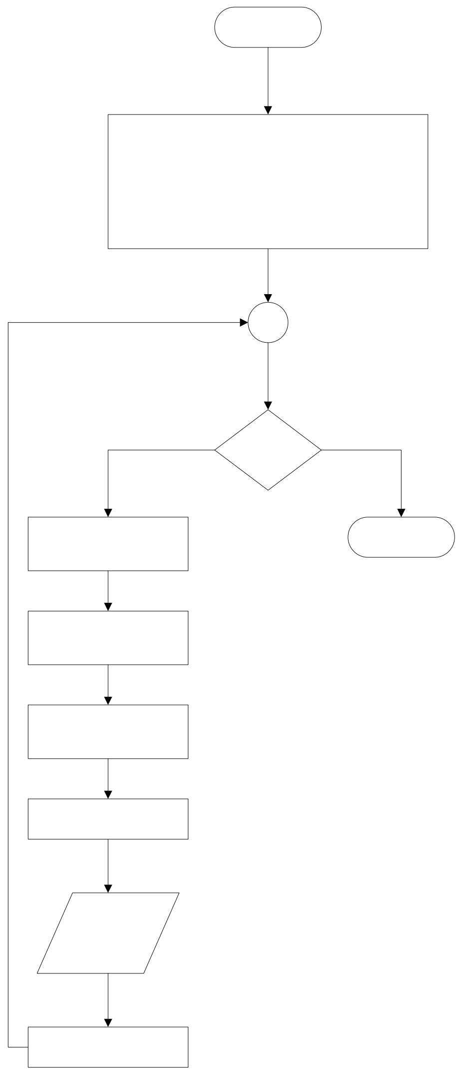
where:

$$u \in \left[-\frac{L}{2}, \frac{L}{2}\right],$$

and L represents the length of the hexagon's edge.

The assembly of the (4) and (5) equations represent the pinion cutter's profile reciprocally enwrapping the polygonal bushing.

The flowchart software for the calculus of the pinion cutter's profile is shown below:



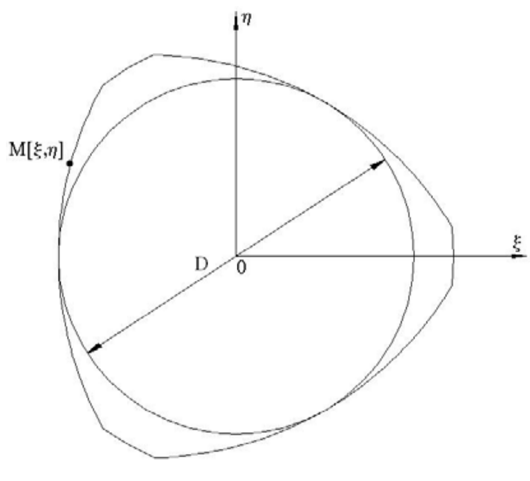


Fig. 2. *The pinion cutter profile*

In figure 2 and table 1 the coordinates of the pinion cutter's profile are shown for the following values of the inscribed circle dimensions: D = 59,58 mm and the transmission ratio i = 6/3.

Table 1

N°	ξ [mm]	η [mm]	u [mm]
1	-20.350	-35.247	-40.700
2	-21.318	-33.515	-38.700
3	-22.223	-31.783	-36.700
4	-23.068	-30.051	-34.700
5	-23.856	-28.319	-32.700
6	-24.589	-26.587	-30.700
7	-25.267	-24.855	-28.700
8	-25.894	-23.123	-26.700
9	-26.471	-21.391	-24.700
10	-26.998	-19.659	-22.700
11	-27.477	-17.927	-20.700
12	-27.909	-16.195	-18.700
13	-28.295	-14.463	-16.700
14	-28.635	-12.731	-14.700
15	-28.931	-10.999	-12.700
⋮	⋮	⋮	⋮
30	-28.184	14.982	17.300
31	-27.784	16.714	19.300
32	-27.338	18.446	21.300
33	-26.845	20.178	23.300
34	-26.303	21.910	25.300
35	-25.712	23.642	27.300
36	-25.069	25.375	29.300
37	-24.375	27.107	31.300
38	-23.626	28.839	33.300
39	-22.821	30.571	35.300
40	-21.958	32.303	37.300
41	-21.034	34.035	39.300

3. Approximated solution for the pinion cutter profile

The approximation of the pinion cutter cross section with a better technological form – a polyform profile machined using a cylindrical enclosing surface, is suggested. [2], [3]

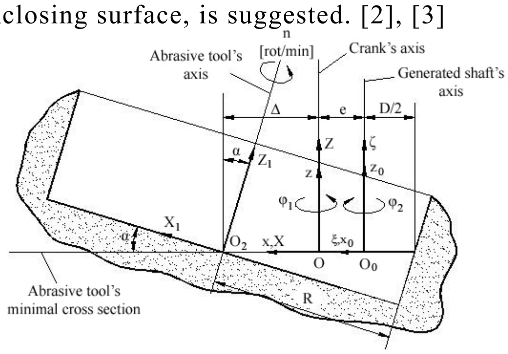


Fig.3. *Reference systems*

Thus, for the generation scheme shown in figure 3, the polyform surface associated to the generated shaft's axis is defined.

For the cross section the following relationships are known:

$$\zeta = H, \text{ with H variable,} \tag{6}$$

$$t = \frac{H + \Delta \sin\alpha}{\cos\alpha} + R tg\alpha \cos\theta \tag{7}$$

The family of surfaces:

$$(C)_{\varphi_1} : \begin{cases} \xi = A(\theta)\cos(\varphi_2 - \varphi_1) - \\ \quad -B(\theta)\sin(\varphi_2 - \varphi_1) + e\cos\varphi_1; \\ \eta = A(\theta)\sin(\varphi_2 - \varphi_1) + \\ \quad + B(\theta)\cos(\varphi_2 - \varphi_1) - e\sin\varphi_1; \\ \zeta = C(\theta). \end{cases} \tag{8}$$

and conditions (4) and (5):

$$\frac{\xi'_{\varphi_1}}{\xi'_\theta} = \frac{\eta'_{\varphi_1}}{\eta'_\theta} \tag{9}$$

In (9) the partial derivatives are defined as:

$$\begin{aligned} \xi'_\theta &= A'(\theta)\cos(i-1)\varphi_1 - \\ &\quad - B'(\theta)\sin(i-1)\varphi_1 - \frac{dt}{d\theta}\sin\alpha; \\ \eta'_\theta &= A'(\theta)\sin(i-1)\varphi_1 + \\ &\quad + B'(\theta)\cos(i-1)\varphi_1 - \frac{dt}{d\theta}\sin\alpha; \\ \xi'_{\varphi_1} &= -(i-1)A(\theta)\sin[(i-1)\varphi_1] - \\ &\quad -(i-1)B(\theta)\cos[(i-1)\varphi_1] - e\sin\varphi_1; \\ \eta'_{\varphi_1} &= (i-1)A(\theta)\cos[(i-1)\varphi_1] - \\ &\quad -(i-1)B(\theta)\sin[(i-1)\varphi_1] - e\cos\varphi_1. \end{aligned} \tag{10}$$

The following values were used for the generation process: R = 150 mm, e = 13,7 mm and α = 0°.

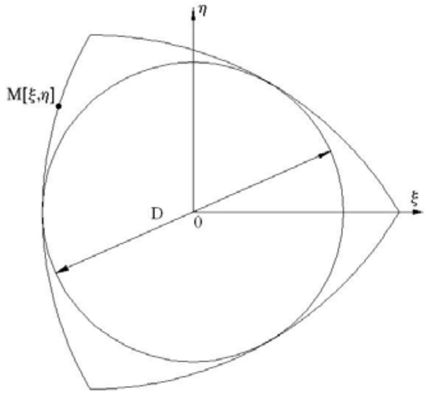


Fig.4. *The grinded shaft profile*

In figure 4 and table 2, the coordinates of the grinded polyform shaft's profile are shown for the following values of the inscribed circle's dimensions: D = 59,58 mm and the transmission ratio, i = 3/4.

Table 2

N°	ξ [mm]	η [mm]
1	-20.607	34.817
2	-21.463	33.252
3	-22.275	31.677
⋮	⋮	⋮
33	-23.075	-30.026
34	-22.308	-31.610
35	-21.498	-33.186
36	-20.643	-34.752

4. Solution for the approximation of the pinion cutter's profile with the polyform profile

The approximation of the two profiles is shown in the following paragraphs. It is obvious that the two profiles aren't identical, the approximation method being affected by errors. The approximation errors are shown in figures 5, 6 and 7.

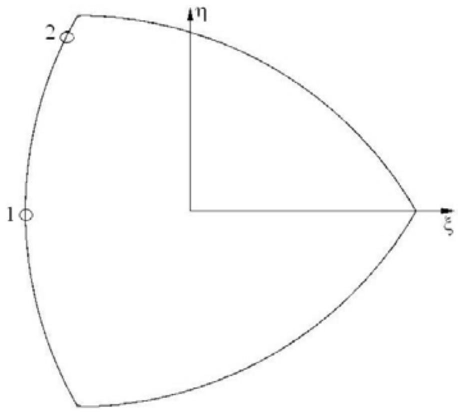


Fig.5. *The approximation of the two profiles*

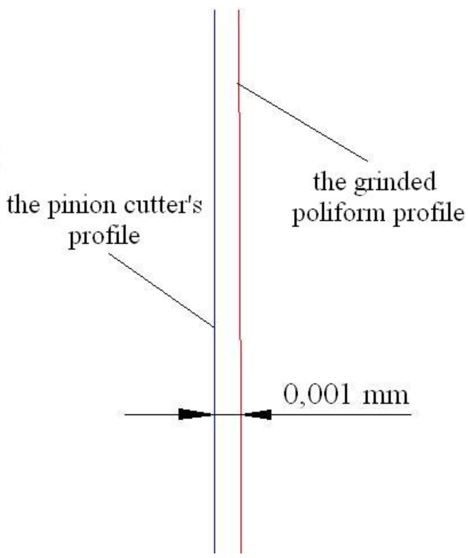


Fig.6. *The error in area 1*

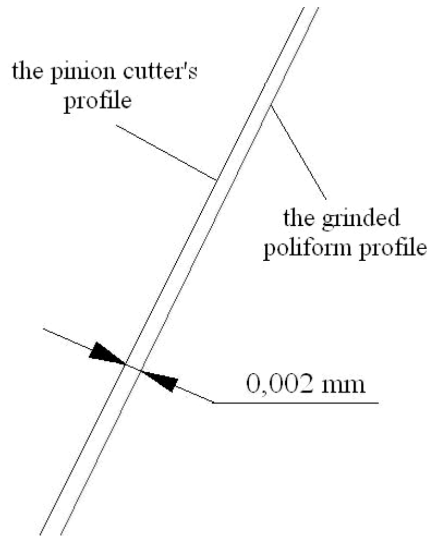


Fig.7. *The error in area 2*

5. Conclusions

The approximation of the pinion cutter using profiles generated using the polyform surfaces technique has shown that this approximation is possible and can be very rigorous. This makes the grinding of the pinion cutter possible after the quenching treatment, when the only available machining process is the grinding process.

This is a general method and it can be applied to polygonal shafts having square or hexagonal sections.

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Aproximarea profilului cuțitului-roată utilizat la generarea suprafețelor poliforme interioare

Rezumat

Sunt cunoscute principiile profilării sculelor de tip roată pentru mortezarea unor profiluri transversale ale butucilor cu forme hexagonale sau pătrate.
În lucrare, se propune o metodologie de aproximare a profilurilor sculelor de tip roată cu profiluri poliforme care se bucură de proprietatea că au o tehnologe de realizare mai simplă, fără a necesita o maşină cu comandă numerică.

L'approximation du Profil de Coupeur de Matériel Utilisé dans la Génération de Surfaces de Poliform Intérieures

Résume

Ils sont connus les principes dressants le portrait pour le coupeur de matériel pour le mortaiser de moyeu traversant des profils avec la section hexagonale ou carrée. Dans ce papier est proposé une nouvelle méthodologie d'approximation pour les profils de coupeur de matériel polyformes qui ont une technologie usinant plus simple, sans besoin une machine contrôlée numérique.