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Study on the Transient Conjugated Heat Transfer in Developing Laminar Pipe Flow

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ABSTRACT

This paper is presenting an analysis of the transient conjugated heat transfer flow in a pipe. The laminar flow and the developing region are considered. The research is oriented towards the influence of the wall pipe thickness and the cooling conditions (Biot number) on the temperature profile, the interface temperature (the interface fluid/pipe wall) and the heat flux at the interface.
Keywords: pipe, laminar flow, heat flux losses.

1. Introduction

Even if the analysis of the laminar pipe flow received great attention in the past, the transient conjugated heat transfer problem for both thermally and hydrodynamically developing laminar pipe flow did not received the right consideration [1]. Nimr & Hader [1] analyzed this particular case of the laminar pipe flow for a Prandtl number Pr=0.7 and the condition of Dirichlet type at the pipe exterior.

This paper follows the non-dimensional procedure used previously [1] and, after rediscovering the Nimr & Hader results, is making a step further in the analysis of this particular process. A more realistic approach is considered for the exterior of the pipe wall taking into account the cooling by convection of the pipe. The Biot number is expressing the degree of cooling conditions. The previous results [1] are obtained for a Biot number of order 1000 which shows that [1] established the upper limit for the cooling conditions and the heat flux from the fluid to the pipe wall.

2. Numerical model

The dimensionless governing equations of the fluid/pipe system were presented elsewhere [1],[2]. Using (U,V) for the velocity in the radial (R) and axial (Z) directions, τ for time, P for pressure, θ_f for the fluid dimensionless temperature, θ_s for the pipe wall temperature, α_R for the ratio of the pipe wall and fluid thermal diffusivities, K_R for the ratio of the pipe wall and fluid thermal conductivities, β for the dimensionless exterior pipe radius and 1.0 for the interior pipe radius, the non-dimensional governing equations are:

- the continuity equation:

$$\frac{\partial U}{\partial Z}+\frac{1}{R}\frac{\partial(RV)}{\partial R}=0\,; \qquad (1)$$

- axial momentum equation:

$$U\frac{\partial U}{\partial Z}+V\frac{\partial U}{\partial R}=-\frac{\partial P}{\partial Z}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right); \qquad (2)$$

- energy equation in the fluid:

$$\frac{\partial\theta_f}{\partial\tau}+U\frac{\partial\theta_f}{\partial Z}+V\frac{\partial\theta_f}{\partial R}=\frac{1}{Pr}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta_f}{\partial R}\right)\right] \qquad (3)$$

- energy equation in the pipe wall:

$$\frac{\partial\theta_s}{\partial\tau}=\frac{\alpha_R}{Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta_s}{\partial R}\right); \qquad (4)$$

- the integral form of continuity equation:

$$\int_0^1 URdR=\frac{1}{2}\,. \qquad (5)$$

The equations (1), (2) and (5) are solved together with the following initial and boundary conditions:

at $\tau\ge 0,\ Z=0\ ,\ 0\le R\le 1$;

$$U=1 \text{ and } V=0; \qquad (6)$$

at $\tau\ge 0,\ Z>0\ ,\ R=1;\ U=V=0;$ (7)

at $\tau\ge 0,\ Z>0\ ,\ R=0;\ \dfrac{\partial U}{\partial R}=\dfrac{\partial V}{\partial R}=0\,.$ (8)

The equations (3) and (4) are solved together with the following initial and boundary conditions:

at $\tau = 0$, $Z \geq 0$ and $0 \leq R \leq \beta$;

$$\theta_s = \theta_f = 1; \tag{9}$$

at $\tau \geq 0$, $Z > 0$ and $R = 1$;

$$\theta_s = \theta_f; \; K_R \frac{\partial \theta_s}{\partial R} = \frac{\partial \theta_f}{\partial R}; \tag{10}$$

at $\tau \geq 0$, $Z > 0$ and $R = 0$; $\frac{\partial \theta_f}{\partial R} = 0$; (11)

at $\tau > 0$, $Z = 0$ and $0 \leq R \leq \beta$;

$$\theta_s = \theta_f = 1; \tag{12}$$

at $\tau > 0$, $Z > 0$ and $R = \beta$; $\theta_s = 0$. (13)

We analyzed not only the Nimr and Hader [1] model but we constructed a model for convective cooling at the pipe exterior boundary. The equation (13) becomes:

at $\tau > 0$, $Z > 0$, $R = \beta$;

$$\frac{\partial \theta_s}{\partial R} + Bi \cdot \theta_s = 0. \tag{14}$$

The numerical method we used for solving this systems of partial differential equations is the finite difference method [3]. The system of equations (1), (2) and (5) was solved using the simple implicit method [3],[4], while the system of equation (3) and (4) was solved using the ADI (Alternating-Direction Implicit) method. A number of 17 points were used in both Z and R direction and the non-dimensional time step was $\Delta\tau = 10^{-5}$ throughout the paper. The first and second order finite differences were centered while for the boundary points, we used forward or backward finite differences.

3. Modeling results

The temperature radial variation is presented for four moments in Fig. 1. This figure is replicating the results obtained in the literature [1] and is validating the numerical model we constructed.

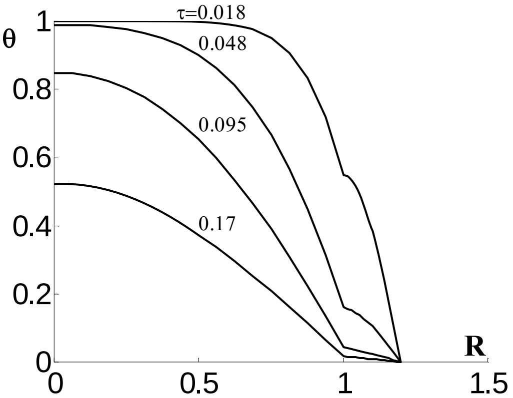

Fig.1 The radial variation of the temperature. Z=0.7; α_R=1.0; β=1.2; K_R=10.

The influence of the cooling conditions on the temperature radial variation is presented by Fig. 2. The difference between Nimr & Hader model [1] and the cooling conditions model, our model, increases as time increases.

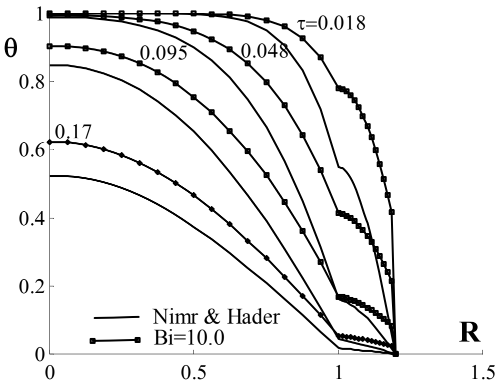

Fig.2 The radial variation of the temperature. Z=0.7 α_R=1.0 β=1.2 K_R=10.

The time variation of the dimensionless heat flux for different cooling conditions (for different Biot numbers) are presented by Fig. 3. A Biot number as high as 1000 is reproducing the results of Nimr & Hader [1] model. A higher Biot number implies a higher heat flux transferred from the fluid to the pipe wall. The condition used by Nimr & Hader is equivalent to the lack of isolation at the pipe wall. For the real situations, the case treated in the literature [1] is providing us the upper level for the heat losses from the fluid transported by a pipe.

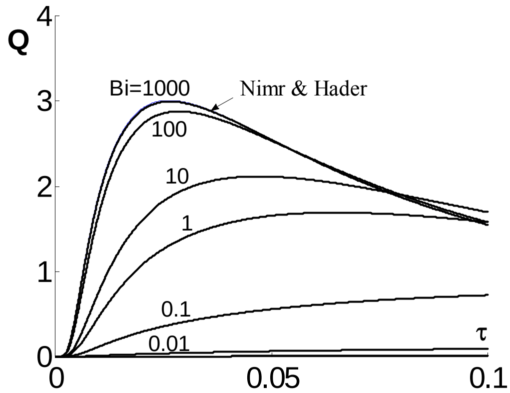

Fig 3 Dimensionless heat flux variation as a function of dimensionless time for different Biot number as well as using Nimr & Hader derivation. Z=0.7 α_R=1.0 β=1.2 K_R=10.

These losses are smaller for a thicker wall pipe. This is demonstrated by Fig. 4 for different ratios of the exterior and interior radius of the pipe, β.

The difference between the values indicated by the literature [1] and our model for the dimensionless heat flux from the fluid to the pipe wall is smaller when the dimensionless exterior radius increases (when the pipe wall is thicker). Fig. 5 is emphasizing the dependence of the dimensionless heat flux from the fluid to the pipe wall for a real range of exterior radius.

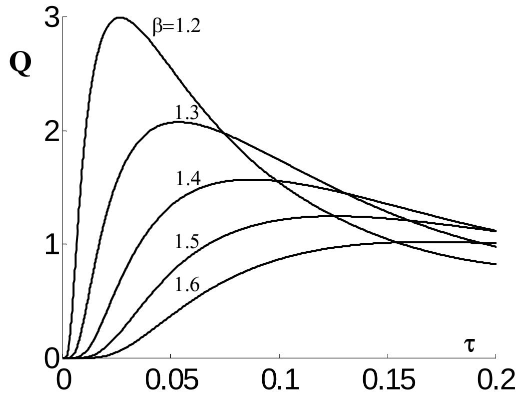

Fig. 4 Heat flux variation as a function of time for different pipe thicknesses; Z=0.7 α_R=1.0 K_R=10 using Nimr & Hader model [1].

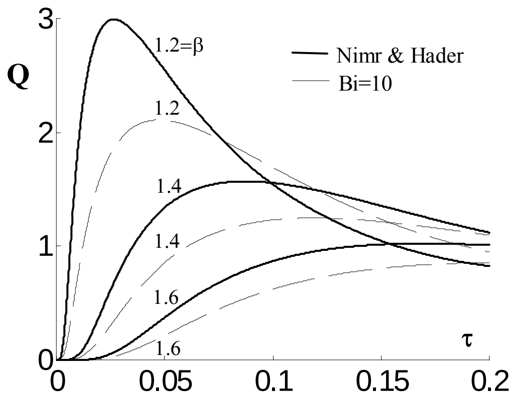

Fig. 5 Heat flux variation as a function of time for different pipe thicknesses. Z=0.7; α_R=1.0; K_R=10.

Another parameter that is influenced by the cooling conditions (Biot number) is the interface temperature θ_i. (Fig. 6). In the early stages of the flowing process, the difference between the values presented by [1] and our model are bigger. The difference becomes smaller at later time.

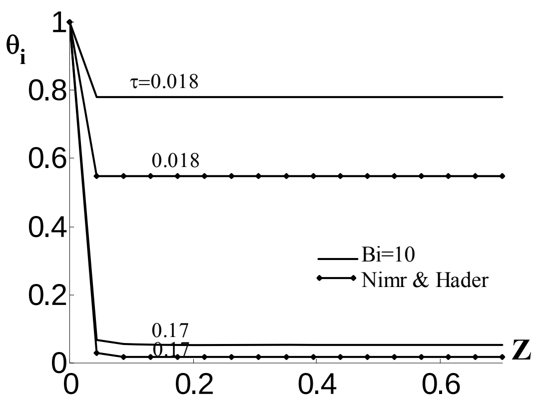

Fig. 6 Interface temperature - Z variation for two moments in time. Z=0.7 α_R=1.0 K_R=10.

Figure 7 presents the interface temperature variation as a function of time. The interface temperature decreases faster for higher Biot number. The maximum decrease corresponds to Nimr & Hader approach.

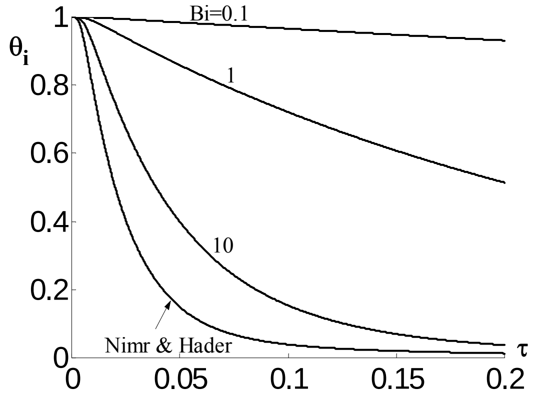

Fig. 7 Interface temperature variation as a function of time direction for different Biot numbers as well as Nimr & Hader case . Z=0.7 α_R=1.0 K_R=10.

4. Conclusions

This paper presents an analysis of the transient heat transfer in the laminar pipe flow, in the developing region. Developing previous research [1], it considers a new approach for the pipe wall external boundary condition improving the modeling results and giving us a better understanding of the thermal transport phenomena during fluid flow.

Our model (Fig.2) is presenting the differences between the two models regarding the temperature field. These differences become bigger as time increases, especially near the pipe axis

The modeling results are emphasizing that smaller Biot numbers and thicker pipe walls determine smaller heat flux losses. The difference between the two approaches becomes greater when the wall pipe thickness is smaller, It means that the difference is important for the real cases.

As a consequence of the phenomena described above, the interface temperature decrease faster when the Biot numbers is bigger, the difference being more important in the early stages of the flow.

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Studiu asupra Transferului de Căldură
la Curgerea Laminară Nedezvoltată prin Conducte

REZUMAT

Acestă lucrare prezintă analiza transferului de căldură la curgerea nestationară într-o conductă. Se consideră cazul curgerii laminare şi a regiunii nedezvoltate. Cercetarea este orientată către studiul influenței grosimii peretelui conductei şi a condițiilor de răcire (numărul lui Biot) asupra profilului temperaturii, temperaturii la interfață (interfața fluid/conductă) şi a fluxului de căldură la interfață.

Untersuchung über das Trtansient Konjugierte Hitze Trnafer,
wenn Blatteriger Rohrfluß Entvickelt Wird

AUSZUG

Dieses Papier stellt eine Analyse des vorübergehenden konjugierten Wärmeübertragungflusses in ein Rohn dar. Der blätterige Fluß wird betrachlet and das Entwicklungs gebiet wird analysiert. Die Forschung wird in Richtung zum Einfluß der Wandrohstärke und der Abkühlbedingungen (Biot Zahl) auf das Temperaturprofil, die Schnittstelle Temparatur (die Schnittstelle fluis/pipe wand) unf den Hitzefluß an der Schnittstelle orientiert.