The Establishment of Compatibility Domain in the Case of Kinematics Groups with Two Common Wheels

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ABSTRACT

The purpose of this paper is to establish the compatibility domain in the cases of transmission mechanisms based on gears with common wheels main types, when these mechanisms are from kinematics groups used to adjust in steps the transmission ratio. Graphic interpretations, drawn in four concrete cases of mechanisms structure, by using a dedicated soft, are presented.

***Keywords**: kinematics groups, gears, common wheels.*

1. General Theoretical Considerations

When designing mechanisms to realize rotation speed step adjustment, an important phase consists in finding kinematics groups gears number of teeth. As consequence of this calculus, it is possible to obtain, on the kinematics group intermediary shaft, gears having the same teeth number.

If the same module value is chosen for both consecutive kinematics groups, one of the wheels can be eliminated in every single case. Thus, common wheels will simultaneously gear with leading wheel from first kinematics group and with driven wheel from the second kinematics group.

By using common wheels, material and manufacturing costs are reduced together with regulation mechanism dimensions decreasing.

Two consecutive kinematics groups can have one or two common wheels, the case with three common wheels being hypothetical.

The commutations number done by kinematics groups with common wheels is

$$q_c = q_{jx_j} \times q_{kx_k}, \qquad (1)$$

where q_j, q_k mean the number of commutations in the cases of “j” or “k” kinematics groups order; x_j, x_k – structure indexes, which, depending on rotation speeds range ratio, φ, gives the kinematics groups ratios:

$$\varphi_j = \varphi^{x_j}. \qquad (2)$$

When designing kinematics groups with a single common wheel there are no special problems; problems appear in the cases of more common wheels, when compatibility domains must be found. This paper wants to establish the compatibility domain in the cases given by two common wheels kinematics groups.

By considering that the mechanisms from the kinematics groups are based on sliding blocks of gears with two or three commutations, the kinematics schemes of four types of mechanisms with two common wheels are presented in Fig.1.

Between the transmission ratios of gears with common wheels, the relations

$$i_{j2} = i_{j1} \cdot \varphi_j;\ i_{k2} = i_{k1} \cdot \varphi_k, \qquad (3)$$

can be written.

If $\sum z_j$ and $\sum z_k$ are the kinematics groups “j” and “k” gears numbers of teeth sums, having in view that, in the case of the common wheels, we can write $z_{j1}' = z_{k2}$, respective $z_{j2}' = z_{k1}$, it results:

$$\sum z_j \cdot \frac{1}{1+i_{j1}} = \sum z_k \cdot \frac{i_{k2}}{1+i_{k2}};$$
$$\sum z_j \cdot \frac{1}{1+i_{j2}} = \sum z_k \cdot \frac{i_{k1}}{1+i_{k1}}. \qquad (4)$$

By eliminating numbers of teeth sums, a relation connecting transmission ratios of gears including common wheels is obtained as

$$\frac{1+i_{j2}}{1+i_{j1}} = \frac{i_{k2}}{i_{k1}} \cdot \frac{1+i_{k1}}{1+i_{k2}}. \qquad (5)$$

The minimum transmission ratios of the kinematics groups multiplication gives the total minimum transmission ratio.

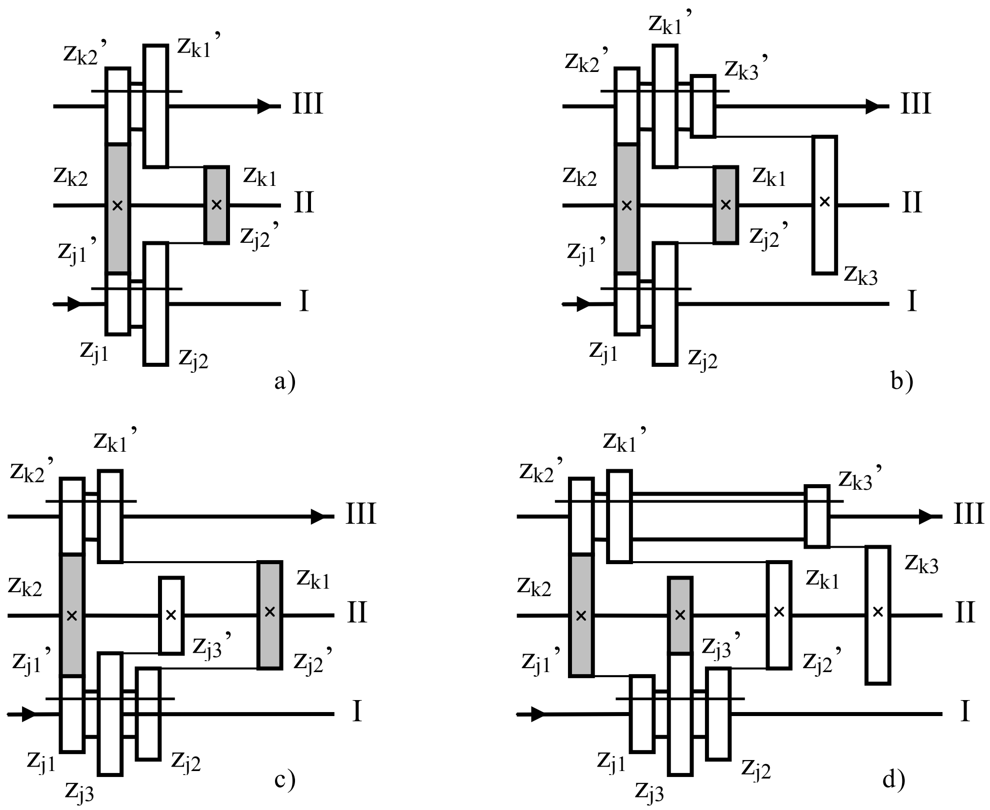

Fig.1 – Kinematics Groups with Two Common Wheels Types of Structure

By denominating this transmission ratio as $i_m = i_{j1} \cdot i_{k1}$, it results

$$i_{k1} = \frac{i_m}{i_{j1}}. \qquad (6)$$

By using also relations (3), from (5) it follows:

$$\frac{1 + i_{j1} \cdot \varphi_j}{1 + i_{j1}} = \frac{\varphi_k \left(i_{j1} + i_m \right)}{i_{j1} + i_m \cdot \varphi_k}. \qquad (7)$$

Based on these relations, the minimum transmission ratio of "j" order kinematics group results as

$$i_{j1} = \frac{\varphi_k - 1 - i_m \cdot \varphi_k \left(\varphi_j - 1 \right)}{\varphi_j - \varphi_k}, \qquad (8)$$

together with others common wheels gears transmission ratios:

$$i_{j2} = \frac{\varphi_k - 1 - i_m \cdot \varphi_k \left(\varphi_j - 1 \right)}{\varphi_j - \varphi_k} \cdot \varphi_j; \qquad (9)$$

$$i_{k1} = \frac{i_m \left(\varphi_j - \varphi_k \right)}{\varphi_k - 1 - i_m \cdot \varphi_k \left(\varphi_j - 1 \right)}; \qquad (10)$$

$$i_{k1} = \frac{i_m \cdot \varphi_k \left(\varphi_j - \varphi_k \right)}{\varphi_k - 1 - i_m \cdot \varphi_k \left(\varphi_j - 1 \right)}. \qquad (11)$$

2. Applications

By using the upper mentioned relations in the case of a four commutations mechanism, $4 = 2_1 \times 2_2$, (Fig. 1-a), when $q_j = 2$, $x_j = 1$, $\varphi_j = \varphi$, $q_k = 2$, $x_k = 2$, $\varphi_k = \varphi^2$ (φ meaning the successive rotation speeds range ratio), the following relations are obtained:

$$\begin{aligned} i_{j1} &= \frac{i_m \cdot \varphi^2 - \varphi - 1}{\varphi}; \\ i_{j2} &= i_m \cdot \varphi^2 - \varphi - 1; \\ i_{k1} &= \frac{i_m \cdot \varphi}{i_m \cdot \varphi^2 - \varphi - 1}; \\ i_{k2} &= \frac{i_m \cdot \varphi^3}{i_m \cdot \varphi^2 - \varphi - 1}. \end{aligned} \qquad (12)$$

By imposing to the kinematics groups minimum and maximum transmission ratios the restrictive conditions:

$$i_{j1} \geq \frac{1}{4},\ i_{j2} \leq 2,\ i_{k1} \geq \frac{1}{4},\ i_{k2} \leq 2,$$

relations from (12) become, respectively:

$$i_m \geq \frac{5 \cdot \varphi + 4}{4 \cdot \varphi^2}; \quad (13)$$

$$i_m \leq \frac{\varphi + 3}{\varphi^2}; \quad (14)$$

$$i_m \geq \frac{\varphi + 1}{\varphi^2}; \quad (15)$$

$$i_m \geq \frac{2(\varphi + 1)}{\varphi^2 (2 - \varphi)}. \quad (16)$$

If in relations (13) ... (16) extreme values are given to φ ratio (φ_{min} = 1, φ_{max} = 2), we can observe that in the case of any value for φ, 1 < φ < 2, the minimum transmission ratio results greater than 1. In reality, the output shaft rotation speeds must be smaller than regulation mechanism input shaft rotation speed, this kind of kinematics group with common wheels hasn't practical interest.

If the succession to generate structure indexes is reversed, the structural formula becomes $4 = 2_2 \times 2_1$, with $q_j = 2$, $x_j = 2$, $\varphi_j = \varphi^2$, $q_k = 2$, $x_k = 1$, $\varphi_k = \varphi$. Thus, based on relation (8) ... (11), we can write:

$$i_{j1} = \frac{1 - i_m \cdot \varphi(\varphi + 1)}{\varphi};$$

$$i_{j2} = [1 - i_m \cdot \varphi(\varphi + 1)]\varphi;$$

$$i_{k1} = \frac{i_m \cdot \varphi}{1 - i_m \cdot \varphi(\varphi + 1)}; \quad (17)$$

$$i_{k2} = \frac{i_m \cdot \varphi^2}{1 - i_m \cdot \varphi(\varphi + 1)}.$$

By imposing the same restrictions as above, on the transmission ratios, it results:

$$i_m \leq \frac{4 - \varphi}{4\varphi(\varphi + 1)}; \quad (18)$$

$$i_m \geq \frac{\varphi - 2}{\varphi^2 (\varphi + 1)}; \quad (19)$$

$$i_m \geq \frac{1}{\varphi(\varphi + 5)}; \quad (20)$$

$$i_m < \frac{1}{\varphi(\varphi + 1)}; \quad (21)$$

$$i_m \leq \frac{2}{\varphi(3\varphi + 2)}. \quad (22)$$

In Fig.2 the curves representing the conditions (18) ... (22) – denominated, respective, from 1 to 5 - are drawn; thus, the minimum transmission ratio compatibility domain, in the case of this structure results, in graphic form.

It follows that, in the case of mechanisms with two common wheels by $q_{jx_j} \times q_{kx_k}$ general structure, it is necessary, in order to assure rotation speed reduction, that the structure index of second kinematics group, x_k, should be $x_j = x_k \cdot q_k$.

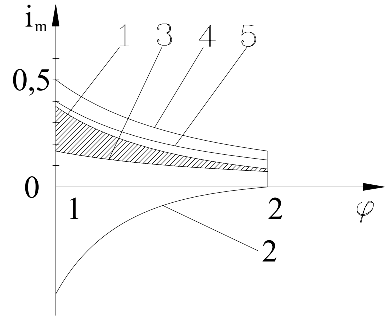

Fig.2 – Compatibility Domain - I

We further find, on the same way, the compatibility domains in the cases of the others types of mechanisms shown in Fig.1.

When mechanism from Fig 1-b, having the structural formula $6 = 2_3 \times 3_1$, with $q_j = 2$, $x_j = 3$, $\varphi_j = \varphi^3$, $q_k = 3$, $x_k = 1$, $\varphi_k = \varphi$ is considered, it results:

$$i_m \leq \frac{4 - \varphi - \varphi^2}{4\varphi(\varphi^2 + \varphi + 1)}; \quad (23)$$

$$i_m \geq \frac{\varphi^2 - 2\varphi - 2}{\varphi^3 (\varphi^2 + \varphi + 1)}; \quad (24)$$

$$i_m \geq \frac{1}{\varphi(\varphi^2 + 5\varphi + 5)}; \quad (25)$$

$$i_m < \frac{1}{\varphi(\varphi^2 + \varphi + 1)}; \quad (26)$$

$$i_m \leq \frac{2}{\varphi(3\varphi^2 + 3\varphi + 2)}. \quad (27)$$

In Fig.3, the compatibility domain results by intersecting the curves drawn based on relations (23) ... (27) – denominated, respective, from 1 to 5.

In the case of the mechanism 1-c from Fig.1, having the structural formula

$6 = 3_2 \times 2_1$, when $q_j = 3$, $x_j = 2$, $\varphi_j = \varphi^2$, $q_k = 2$, $x_k = 1$, $\varphi_k = \varphi$, the same compatibility domain as in the case of 1-a mechanism, upper solved, results.

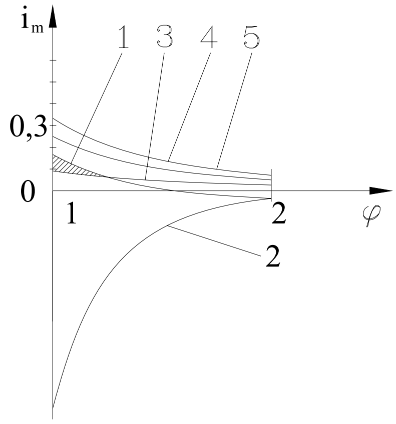

Fig.3 – Compatibility Domain – II

Finally, when mechanism from Fig 1-d, having the structural formula $9 = 3_3 \times 3_1$, with $q_j = 3$, $x_j = 3$, $\varphi_j = \varphi^3$, $q_k = 3$, $x_k = 1$, $\varphi_k = \varphi$ is considered, it results the same compatibility domain as shown in Fig.3.

In the cases upper analyzed, the supposition that the kinematics groups with common wheels are disposed at the gearbox input was made. In fact, these kinematics groups can occupy any place into the regulation mechanism structure. For example, in the case of a $12 = 3_1 \times 2_6 \times 2_3$ structure, when kinematics groups with common wheels are giving the $2_6 \times 2_3$ part, the conditions to be imposed to the minimum transmission ratio are:

$$i_m \le \frac{4-\varphi^3}{4\varphi^3(\varphi+1)}; \qquad (28)$$

$$i_m \ge \frac{\varphi^3-2}{\varphi^6(\varphi^3+1)}; \qquad (29)$$

$$i_m \ge \frac{1}{\varphi^3(\varphi+5)}; \qquad (30)$$

$$i_m < \frac{1}{\varphi^3(\varphi^3+1)}; \qquad (31)$$

$$i_m \le \frac{2}{\varphi^3(3\varphi^3+2)}. \qquad (32)$$

In Fig.5, the compatibility domain in this last case is presented.

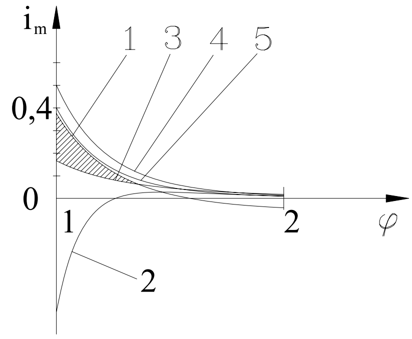

Fig.4 – Compatibility Domain - III

3. Conclusions

The compatibility domain of mechanisms with two common wheels is larger in the case of $2_2 \times 2_1$ structures than in the case of $2_3 \times 3_1$ structures. Same time, the compatibility domain decreases when the difference between structure indexes increases.

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Rezumat

Lucrarea de față își propune să stabilească domeniile de compatibilitate pentru principalele tipuri de mecanisme cu roți dințate comune, mecanisme ce alcătuiesc grupe cinematice din structurile mecanismelor pentru reglarea în trepte a rapoartelor de transmitere. Interpretări grafice, în patru cazuri concrete sunt, de asemenea, prezentate.

Résume

Le but de ce papier est d'établir les domaines de compatibilité pour les principaux types de mécanismes aux roues dentées communes, mécanismes qui constituent des groupes cinématiques dans les mécanismes utilisées pour régler les rapports de transmission. Des interprétations graphiques, dans quatre situations concrètes sont aussi présentées.