

The Theory of Unary Gears

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Abstract

This work presents at the beginning the fundamental gear theorem (Willi's theorem) for the case where the gearing pole (pitch point) is situated at infinity. In this situation, the possibility of a new gear type, the unary gear, is a result. The name is supported by the fact that these gears are the only gear types, which present a transmission ratio of +1. In other words, the gears rotate with the same speed, in the same direction, not only in the interior but also in the exterior gearing!

Keywords: unary gear, new gear type, fundamental gear theorem.

1. Introduction

Although many researchers have approached the gearing theory, the authors of this paper consider that there is still something to add for a particular case as described below.

The fundamental law of gearing shows that two gears yield a constant transmission ratio if the normal common in the contact points passes all through a fixed point, called the pitch point.

The external gears, the Fig. 1, have the pitch point on the line of the rotation centers, between them. The internal gears, the Fig. 2, have their pitch point also on the line of the rotation centres line, but outside them.

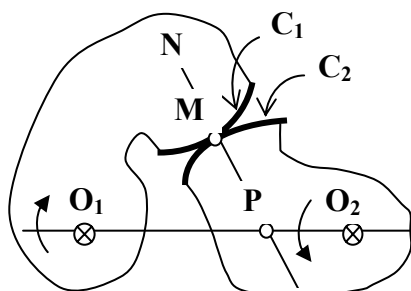


Fig. 1

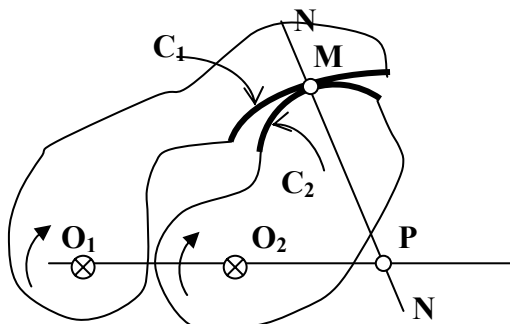


Fig. 2

If there are gears of constant transmission ratio having their pitch point anywhere on the rotation centres line, then gears having their pitch point at the extreme position at infinite also must exist.

The paper attempts to show that such gears exist and to describe their main characteristics.

2. The theory of unary gears

In the paper is considered the case of plane gearing, because starting from this particular case the spatial gearing problem is easy to extrapolate.

Let us assume that there is a plane gear this has the pitch point, at infinite. Let C_1 and C_2 be two curves, which can be used as the gear flanks (the Fig 3).

Since the pitch point is at infinite, it follows that the pitch radii are infinite too. It also results that the rotation directions cannot be but the same. Moreover, the limit of the pitch radii ratio tends to **one**, because:

$$\lim_{\infty} Rr_1 / Rr_2 = \lim_{\infty} (Rr_2 + d) / Rr_2 = 1 + \lim_{\infty} d / Rr_2 = 1.$$

It follows that the transmission ratio i is:

$$i = Rr_2 / Rr_1 = \omega_1 / \omega_2 = +1.$$

Remark: The gears, which have the pitch point at infinite, are the only gears having the transmission ratio equal to +1. The above consideration makes us to suggest, in the paper, to call these gear **unary gears**.

In order to determine what curves can be used as flanks for the unary gears, use the Fig. 3. Let d be the distance between the rotation

centres O_1 and O_2 . If a shifting motion along the O_1 - O_2 direction of the centres line, equal to d , is performed we will find that the corresponding contact points M_1 and M_2 of the curve C_1 and C_2 are located at distance d . The above case is a random one, and, therefore, for

any other position we will find the same. It was shown that the two curves used as flanks for the unary gear are not uniquely determined and they can be any equidistant (parallel) curves having the distance length between them equal to the distance d between the rotation centres.

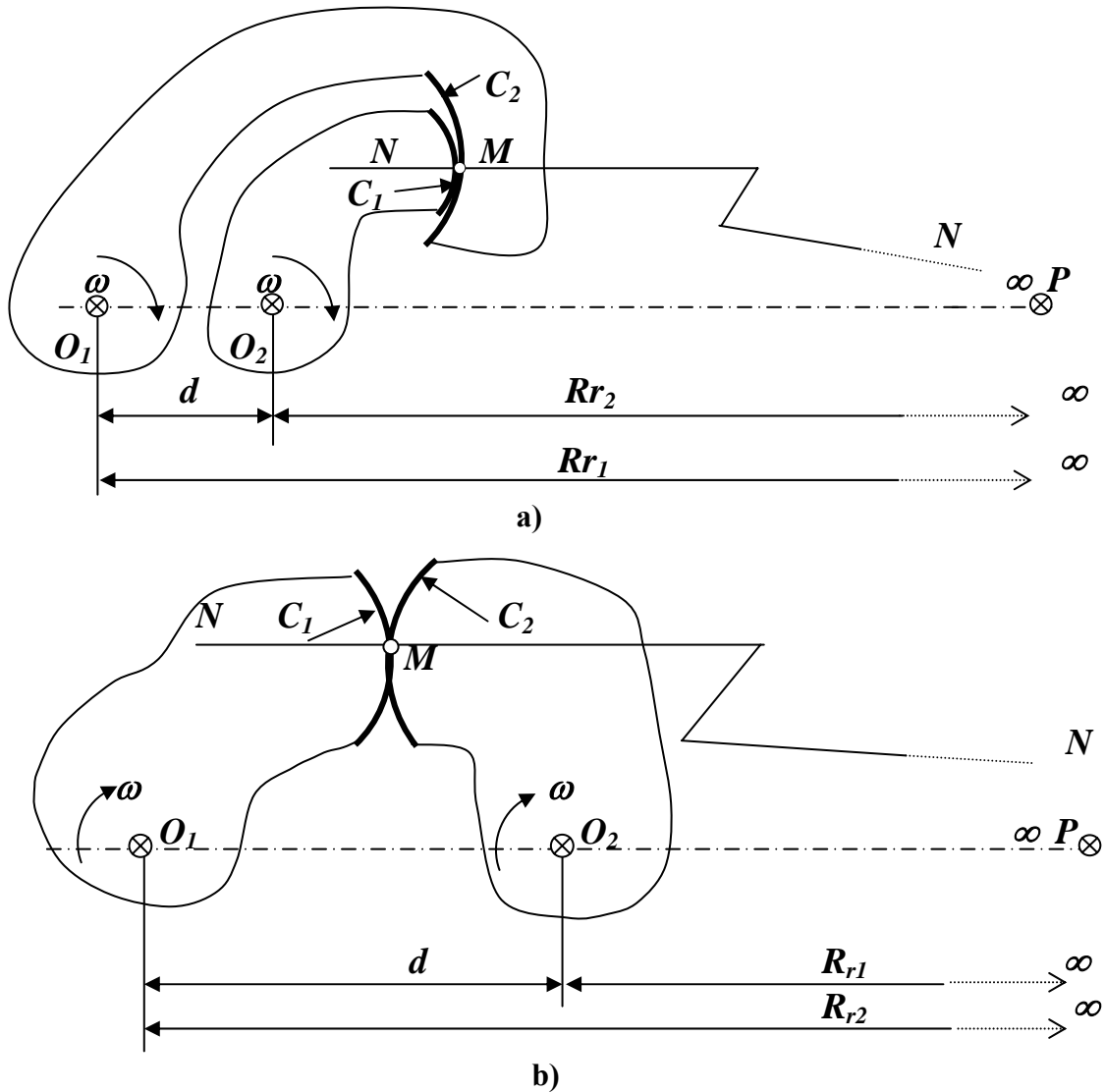


Figure 3

The above demonstration can be summarized into the following:

Theorem: *the corresponding flanks of a unary gear must be any two equidistant (parallel) curves having their equidistance equal to the distance between the rotation centres.*

Remark:

a) *In the case of the external unary gears, the corresponding flanks must be two equidistant (parallel) curves having opposite curvatures, therefore is drawn on*

both sides of their common evolute's, the Fig.4a;

b) *In the case of the internal unary gears, the corresponding flanks must be two equidistant (parallel) curves having their curvatures of the same sign, which is drawn only in one side of their common evolute's, the Fig.4b.*

Some comments are necessary here. Let C is a random curve, locally represented in the vicinity of a point M on the curve, the Fig.5. Let ρ be the curvature radius in M and O the curvature center.

According to **Leibnitz’s** definition:
 If M is a variable point on C (the Fig. 5), the locus of point that lie a distance $\pm e$ units from M along a line perpendicular to C define **equidistant** (parallel) curves to C .

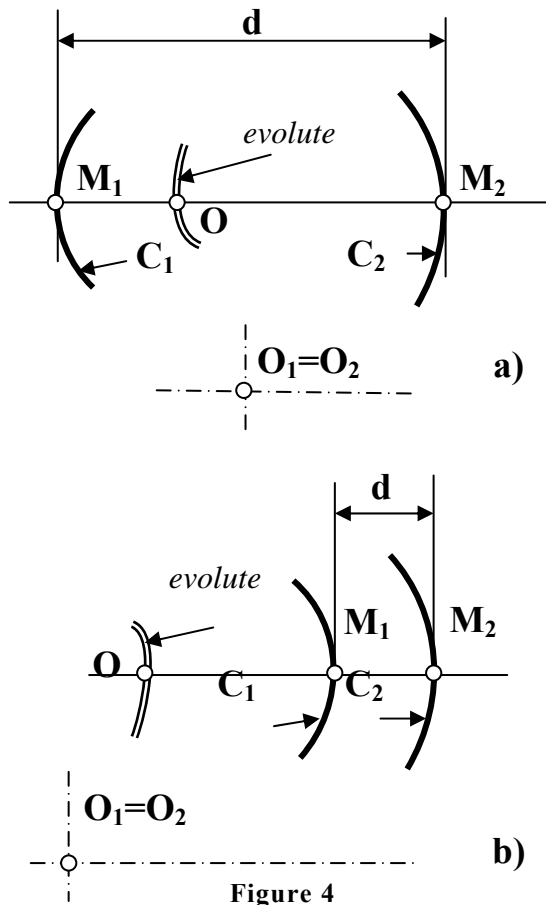


Figure 4

Some comments are necessary here. Let C is a random curve, locally represented in the vicinity of a point M on the curve, the Fig.5. Let ρ be the curvature radius in M and O the curvature center.

According to **Leibnitz’s** definition:
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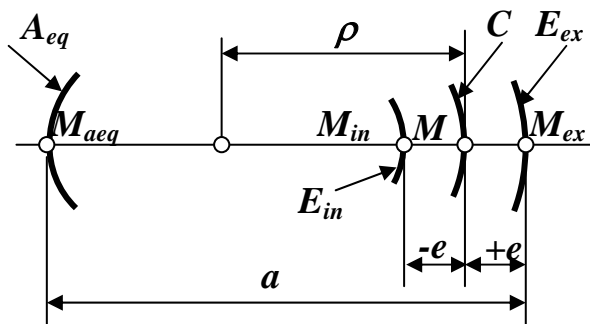


Figure 5

According to the above definition there is no answer to the two following questions:

- For which of the two branches is the equidistante reasonably considered positive or negative?

- What is the drawn curve, also at a constant distance put beyond the curvature center O ?

Without going into details (this will be discussed in another paper), we suggest the following:

- the equidistant curve obtained of increased curvature radius should be called **external equidistant** E_{ex} , of **equidistante** $+e$;
- the equidistant curve obtained of decreased curvature radius should be called **internal equidistant** E_{in} , of **equidistante** $-e$;
- The “internal equidistant” curve obtained of decreased flow curvature radius by $a > \rho$ (therefore a negative curvature radius, opposed to C) should be called **antiequidistant** A_{eq} , of **antiequidistante** a .

Because of the foregoing it follows that, with external unary gears, the corresponding flanks should be antiequidistant curves.

As for as the unary gears are conceived we recommend:

- if the distance between the rotation centers must be small, the internal gear solution s will be found;

- if the distance between the rotation centers must be great, an external gear should be a good choice, because the size of the antiequidistance can be however big and therefore allows for such solutions.

3. Applications of the unary gear theory

Such applications can be found by going through the following steps:

Step I: the shape of one of the gear element is found;

Step II: the equidistant shape (or antiequidistante) corresponding to the shape adopted for the first element by selecting an equidistant equal to the necessary distance between the rotation centres is then found.

Step III: the two shapes matching is checked by performing a shifting translation motion with the equidistance (or antiequidistante).

4. Conclusions

The paper only provides for a brief description of the unary gears. A number of examples easy to understand were given. Theoretically, the

paper is significant by adding information to the general theory of gearing. The considerations advanced in the paper can be practically applied in the field of mechanical transmissions, machining of cylindrical surfaces, for example, by slotting with pinion type cutter.

References

1. **GHIȚĂ Eugen, GHIȚĂ Daniel**, *Unary gears*, In Proceeding, Vol. 1/3 of "4-th World Congress on Gearing and Power Transmission", 16-18 march 1999, Paris, pag. 937-945;

Teoria angrenajelor unare

Rezumat

În lucrare se definește noțiunea "angrenaje unare". Acestea sunt cupluri de două corpuri (piese, roți dințate) ce se rotesc în același sens, cu aceeași viteză unghiulară, având permanent contact. Alegerea acestei denumiri s-a făcut din considerentul că numai asemenea angrenaje permit realizarea unor rapoarte de transmisie egale cu unitatea, respectiv +1. Se demonstrează că, în acest caz, polul angrenării este la infinit și razele cercurilor de rulare sunt infinite.

La théorie des engrenages unitaires

Sommaire

Ce travail présente au début le théorème fondamental de vitesse (le théorème de Willi) pour le cas où le poteau d'embrayage (point de lancement) être situé à l'infini. Dans cette situation, la possibilité d'un nouvel à engrenages, les *engrenages unitaires*, est un résultat. Le nom est soutenu par le fait que ces vitesses sont les seuls types de vitesse, qui présentent un rapport de transmission de +1. En d'autres termes, les vitesses tournent avec la même vitesse, dans la même direction, non seulement dans l'intérieur mais également dans l'embrayage extérieur !