THE ANNALS “DUNĂREA DE JOS” OF GALAȚI
FASCICLE V, TECHNOLOGIES IN MECANICAL ENGINEERING, ISSN 1221-4566
2005

Analytical Model of Worm Hob Errors with Continuous Sharpening

Prof. Dr. Ing. Nicu C•p••în•
University „Dun•rea de Jos” of Gala•i

SUMMARY

The module worm hob, through its construction and positioning on the cutter arbor, is affected by errors which step in the generation process on the involute tooth’s flank of the manufactured gears.

This paper shows the method for determining the errors which appear at the sharpening of the Fredascon module worms hob with positive rake angle, abraded through continuous helical sharpening of the back edge. The errors obtained for the determining of the profiling errors have a general form and can be singularized for all the worm wheel hobs usually used for gear generation.

Keywords: *worm cutter tool, Fredascon*

1. Introduction

To obtain a correct profile on the involute flanks of the machined gear’s teeth, the worm wheel hob’s cutting edges, obtained through the intersection of the locating surfaces of the teeth and of the helical channels for chip’s gap.

By means of practical reasons, the builders of gearing tools consider the outer surfaces of the worm wheel hob as being helical surfaces such as archimedic or convolute types, and the helical channels for chip’s draw off have a null back rake angle. In addition, the radial backing-off clearance on a screw of the module worms hob is usually done with abrasive tools with a classic shape (conical, pot shaped, etc.), the profile resulting with a few approximations [1].

The profiling of the teeth through continuous helical sharpening can be applied for the newest types of Fredascon module worms hob, which have individual teeth, eccentrically fixed towards the axial plane of the tool’s body.

Because of this special construction, the hob’s teeth, fixed in conical holes, placed on the directing helix of the worm tool, can be rotated around their axes to ensure positions needed for the profiling operation using the continuous sharpening of the back edges or for machining.

2. Calculus of the errors that appear with the profiling of worm wheel hobs using continuous sharpening

In the following text, the method of determining the errors that appear at the profiling of Fredascon module worms hob with positive rake angle, ground through continuous helical sharpening on the back edges, is presented.

In general, when the enwrapping worm and the gap surface are ruled helical surfaces, the equations of the primary outer helical surface, to which the cutting edges of the worm tool’s teeth belong in the machining position, become:

$$\Sigma_{1,2}\begin{cases} x = \sqrt{R_0^2 + u^2 \cdot sin^2\beta_0} \cdot cos\,\varphi; \\ y = \sqrt{R_0^2 + u^2 \cdot sin^2\beta_0} \cdot sin\,\varphi; \\ z = \mp p \cdot arctg\left(\dfrac{u \cdot sin\beta_0}{R_0}\right) \pm u \cdot cos\,\beta_0 + p \cdot \varphi, \end{cases} \tag{1}$$

where:

p is the helical parameter of the worm tool’s primary outer surface;

R_0 – the radius of the helical surface’s directing cylinder;

β_0 – the generating line’s angle towards the axial direction;

u and φ - variable parameters of the helical surface.

The errors which appear at the module worms hob machined using the method shown above, will be determined on the incident direction to the theoretically exact enwrapping worm’s involute helical surface.

If the enwrapping worm’s surface, given by the (1) relationship corresponding to the helix’s right flank, is cut off with the (P) plane, tangent to the base cylinder of the involute worm, the (C) curve is obtained showing that the deformation of the real profile compared to the one of the theoretically exact profile (the Δ_d line).

From figure 1, the helix’s angle on the generating line’s base cylinder and the profiling errors on the ε_p^d and ε_p^s flanks can be obtained:

$$tg\omega_b = \frac{Z_2 - Z_1}{X_1 - X_2} = \frac{p}{R_b}; \tag{2}$$

$$\varepsilon_p^d = (Z_3 - Z_1)\cdot cos\,\omega_b - (X_1 - X_3)\cdot sin\,\omega_b; \tag{3}$$

$$\varepsilon_p^s = (Z_1 - Z_3)\cdot cos\,\omega_b - (X_1 - X_3)\cdot sin\,\omega_b, \tag{4}$$

Where:

X_1, X_2, Z_1 and Z_2 are the coordinates of the Δ_d generating line’s intersection points, 1 and 2, of the involutes worm, with the (C) curve;

R_{ib} – the radius of the involutes worm’s base cylinder.

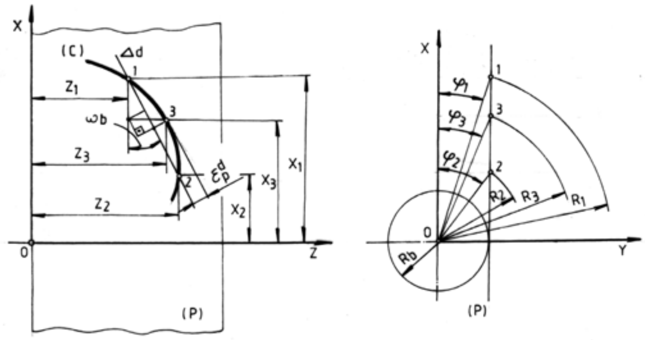


Fig. 1. The deformation of the enwrapping worm’s real profile

The (3) and (4) relations allow the profiling errors on the teeth’s two flanks to be determined in the general case.

Singularity, the profiling errors can be calculated in the case of hobs with generating worms of archimedic (ZA) or convolute (ZN1) and (ZN2) types.

2.1. The calculus of the profiling errors for the continuous sharpening of archimedic (ZA) type module worms hob

The surface of the archimedic type enwrapping worm (figure 2) is determined by the following equations:

$$\Sigma_{1,2}\begin{cases} x = u\cdot cos\,\alpha_d\cdot cos\,\varphi; \\ y = u\cdot cos\,\alpha_d\cdot sin\,\varphi; \\ z = \mp\dfrac{s_{d0}}{2} \mp R_d\cdot tg\alpha_d \pm u\cdot sin\,\alpha_d + p\cdot\varphi. \end{cases} \tag{5}$$

The profiling error on the right flank is determined by the relationship:

$$\begin{aligned}\varepsilon_p^d = {} & [(R_1 - R_3)\cdot tg\alpha_d - \\ & - p\cdot(arcsin\frac{R_b}{R_1} - arcsin\frac{R_b}{R_3})]\cdot cos\,\omega_d \\ & -[\sqrt{R_1^2 - R_b^2} - \sqrt{R_3^2 - R_b^2}\,]\cdot sin\,\omega_d.\end{aligned} \tag{6}$$

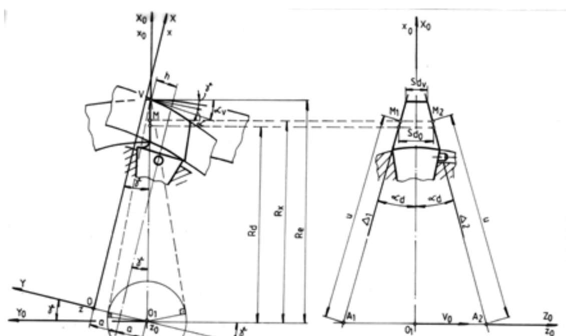


Fig. 2. The continuous generation of helical archimedic surfaces

Similarity, the profiling error on the left flank is determined:

$$\varepsilon_p^s = [(R_1 - R_3)\cdot tg\alpha_d + \\ + p\cdot(arcsin\frac{R_b}{R_1} - arcsin\frac{R_b}{R_3})]\cdot cos\,\omega_d - \\ -[\sqrt{R_1^2 - R_b^2} - \sqrt{R_3^2 - R_b^2}\,]\cdot sin\,\omega_d . \qquad (7)$$

2.2. The calculus of the profiling errors for the continuous sharpening of convolute (ZN1) type module worms hob

The primary outer surface's equations of the convolute (ZN1) worm wheel hob are:

$$\Sigma_{1,2}\begin{cases} x = u(cos\,\alpha_{dn}\,cos\,\varphi \pm sin\,\alpha_{dn}\,sin\,\omega_d\,sin\,\varphi) \mp (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d\,sin\,\varphi; \\ y = u\left(cos\,\alpha_{dn}\,sin\,\varphi \mp sin\,\alpha_{dn}\,sin\,\omega_d\,cos\,\varphi\right) \mp (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d\,cos\,\varphi; \\ z = \pm u\,sin\,\alpha_{dn}\,cos\,\omega_d \mp (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,cos\,\omega_d + p\varphi. \end{cases} \qquad (8)$$

The profiling errors on the teeth's two flanks are obtained with the relations:

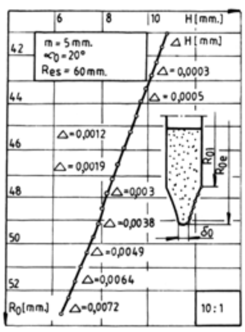
$$\varepsilon_p^d = \frac{-R_b[R_bR_3 + \sqrt{R_3^2 - R_b^2}(\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d]\,sin\,\alpha_{dn}\,cos\,\omega_d}{\sqrt{p^2 + R_b^2}(R_b\,cos\,\alpha_{dn} + \sqrt{R_3^2 - R_b^2}\,sin\,\alpha_{dn}\,sin\,\omega_d)} + \\ + \frac{R_b[R_bR_1 + \sqrt{R_1^2 - R_b^2}(\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d]\,sin\,\alpha_{dn}\,cos\,\omega_d}{\sqrt{p^2 + R_b^2}(R_b\,cos\,\alpha_{dn} + \sqrt{R_1^2 - R_b^2}\,sin\,\alpha_{dn}\,sin\,\omega_d)} + \\ + \frac{pR_b}{\sqrt{p^2 + R_b^2}}(arcsin\frac{R_b}{R_3} - arcsin\frac{R_b}{R_1}) - \frac{p\left(R_3 - R_1\right)}{R_1R_3\sqrt{p^2 + R_b^2}}(\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d - \\ - \frac{p[R_bR_1 + \sqrt{R_1^2 - R_b^2}(\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d][\sqrt{R_1^2 - R_b^2} - R_b\,sin\,\alpha_{dn}\,sin\,\omega_d]}{\sqrt{p^2 + R_b^2}(R_bR_1\,cos\,\alpha_{dn} + R_1\sqrt{R_1^2 - R_b^2}\,sin\,\alpha_{dn}\,sin\,\omega_d)} + \\ + \frac{p[R_bR_1 + \sqrt{R_3^2 - R_b^2}(\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn})\,sin\,\omega_d][\sqrt{R_3^2 - R_b^2} - R_b\,sin\,\alpha_{dn}\,sin\,\omega_d]}{\sqrt{p^2 + R_b^2}(R_bR_3\,cos\,\alpha_{dn} + R_3\sqrt{R_3^2 - R_b^2}\,sin\,\alpha_{dn}\,sin\,\omega_d)}; \qquad (9)$$

$$
\begin{aligned}
\varepsilon_p^s = & \frac{R_b [R_b R_3 + \sqrt{R_3^2 - R_b^2} (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn}) \sin\omega_d] \sin\alpha_{dn} \cos\omega_d}{\sqrt{p^2 + R_b^2} (R_b \cos\alpha_{dn} + \sqrt{R_3^2 - R_b^2} \sin\alpha_{dn} \sin\omega_d)} - \\
& - \frac{R_b [R_b R_1 + \sqrt{R_1^2 - R_b^2} (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn}) \sin\omega_d] \sin\alpha_{dn} \cos\omega_d}{\sqrt{p^2 + R_b^2} (R_b \cos\alpha_{dn} + \sqrt{R_1^2 - R_b^2} \sin\alpha_{dn} \sin\omega_d)} - \\
& - \frac{pR_b}{\sqrt{p^2 + R_b^2}} (\arcsin\frac{R_b}{R_3} - \arcsin\frac{R_b}{R_1}) - \frac{p\left(R_3 - R_1\right)}{R_1 R_3 \sqrt{p^2 + R_b^2}} (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn}) \sin\omega_d - \\
& - \frac{p [R_b R_1 + \sqrt{R_1^2 - R_b^2} (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn}) \sin\omega_d] [\sqrt{R_1^2 - R_b^2} - R_b \sin\alpha_{dn} \sin\omega_d]}{\sqrt{p^2 + R_b^2} (R_b R_1 \cos\alpha_{dn} + R_1 \sqrt{R_1^2 - R_b^2} \sin\alpha_{dn} \sin\omega_d)} + \\
& + \frac{p [R_b R_1 + \sqrt{R_3^2 - R_b^2} (\frac{S_{d_0n}}{2} + R_d tg\alpha_{dn}) \sin\omega_d] [\sqrt{R_3^2 - R_b^2} - R_b \sin\alpha_{dn} \sin\omega_d]}{\sqrt{p^2 + R_b^2} (R_b R_3 \cos\alpha_{dn} + R_3 \sqrt{R_3^2 - R_b^2} \sin\alpha_{dn} \sin\omega_d)} .
\end{aligned} \tag{10}
$$

3. Experimental results

In figures 3 and 4 the form and coordinates of the axial section of second-order tools are presented, and the ε_H error of the profile obtained using the approximate method compared to the classic one for a numerical calculus example is shown too. It’s obvious that the profiling error is very small (under 0,01 mm).

The results of the calculus for the profiling of the Fredascon archimedic worm wheel hob, presented in figure 1, show that the profile’s error determined through the shown method is very small (under 0,01 mm) and may be acceptable from the technical point of view.



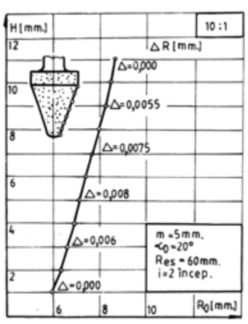


Fig. 3. The disc tool’s profile
Fig. 4. The mil-tool’s profile

4. Conclusions

1. The numerical modeling of the worm tool’s errors which appear in the generation process of the involute flank of the machined gears’ teeth allows the determination of the gearing precision.

2. The shown relations for the profiling errors’ determination have a general form, and can be singularised for the worm wheel hobs usually used for gearing.

3. The calculus results for the profiling of the Fredascon archimedic worm hob show that the error of the determined profile using the method presented above is very small and may be acceptable from the technical point of view.

Bibliography

1. **Căpăţînă, N.,** *Contribuţii la perfecţionarea constructiv-funcţională a frezelor melc modul cu dinţi decalaţi*. Teză de doctorat, Universitatea “Dunărea de Jos” din Galaţi, 1998.

2. **Căpăţînă, N., Ţâru, E.,** *Special Gear Cutting Tools*. În Analele Universităţii din Galaţi, fasc. V, 1995, p. 27-33.

3. **Căpăţînă, N., Ţâru, E.,** *Second-Order Tool Grinding for Continuous Sharpening of the Hobb Cutter Mills Type “Fredascon”*. În Analele Universităţii “Dunărea de Jos” din Galaţi, fasc. V, 1997, p. 23-30.

4. **Căpăţînă, N.,** *Freze melc cu ascuţire continuă*, ISBN 973-731-005-5. Editura Cartea Universitară, Bucureşti, 2004.

MODEL ANALITIC AL ERORILOR FREZEI MELC MODUL CU ASCUŢIRE CONTINUĂ

Rezumat

Freza melc modul, atât prin construcţie cât şi prin modul de poziţionare pe dornul port-sculă este afectată de erori care se transpun în procesul de generare pe flancul evolventic al dinţilor roţilor prelucrate.

În lucrare se prezintă metoda de determinare a erorilor care apar la profilarea frezelor melc modul tip Fredascon cu unghi de degajare pozitiv, rectificate prin ascuţire continuă elicoidală pe feţele de aşezare. Relaţiile obţinute pentru determinarea erorilor de profilare au o formă generalizată şi pot fi particularizate pentru toate cazurile frezelor melc folosite uzual la danturare.

MODÈLE D'ANALITICAL DES ERREURS DE FRAISE-MÈRE DE VER AVEC L'AFFILAGE CONTINU

Résumé

La fraise-mère de ver de module, par sa construction et le positionnement sur l'axe du pourte-outil, est affectée par les erreurs qui sont transmises au procédé de génération sur le flanc en évolvente dents de la rove conpente.

Cet article montre la méthode nécessaire pour déterminer les erreurs ce qui apparaissent à l'affilage de la fraise-mère de vers de module type Fredascon avec angle positif de râteau, la rectification par l'affilage hélicoïdal continu du bord arrière. Les rélations obtenues pour la détermination des erreurs de profilage ont une forme généralisée et peuvent être utilisé pour tontes cas particulièrs de types de fraises-mères de roue de ver habituellement utilisées pour la génération des pièces déntées.