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# THE VIRTUAL POLE METHOD –AN ALTERNATIVE METHOD FOR PROFILING TOOLS WHICH GENERATE BY ENWRAPPING

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## ABSTRACT

*In the paper is presented a method for profiling of tools that generate by enwrapping, by the rolling method, curls ordered by surfaces. By the new method is intended to eliminate one of the main disadvantages associated with the processing by generating of parts with radial symmetry, namely the difficulty related to the calculation of generating tool profile. The method allows eliminating the need to determine analytically the relative movement between the tool and the blank, which simplifies the calculation mode.*

**KEYWORDS:**profiling of tools, gear shaped cutter, rotary cutter, generating by enwrapping.

## 1. INTRODUCTION

In the industrial field, pieces with radial symmetry are often used [1], [2].

The active surfaces of some such pieces, for example the flanks of the teeth or the splined shaft, form so-called curls ordered by surfaces [3].

The processing of curls ordered by surfaces can be done by copying or by generating.

In the case of machining by copying, the axial section of the generating tool materializes the gap that form between two active, anti-homologous surfaces of the curl ordered by surfaces, for example the gap between two teeth of a spur gear.

In the case of processing by generating, the primary peripheral surface of the generating tool is reciprocally enveloping to the surface to be generated, respectively to the side of the tooth, if we consider the previous example.

The machining of surfaces by generating has the advantage of a better precision from the point of view of the surface shape to be obtained and also of some forms of sections of the detached chips favourable to the cutting process. This is due to the fact that the generated surface is processed by a larger number of cuts.

The disadvantage of the machining by generating is given by the necessity of profiling the active surface of the cutting tool, which can lead to relatively difficult calculations.

The processing by generating is done with tools that generate by enwrapping, by the rolling method [4]. This means that between the two conjugated centrodes (of the tool and the piece) there is a rolling motion and the active surfaces of the tool and the piece are in reciprocal enwrapping.

The typical tools used for generating by enwrapping by the rolling method are rack, gear shaped cutter or rotary cutter type.

In the case of any of the three types of tools, determining the active profile of the tool involves following an algorithm which, classically consists in 6 steps [3]:

1. Establishing the analytical equations of the generated profile or determining, by measurement, the coordinates of some points belonging to this profile. The second case, the determination of the coordinates of the points, is used in the case of the use of the reverse engineering techniques, for the reproduction of some existing pieces, whose form is not known by analytical equations.

In either case, the profile to be obtained is expressed in the part's own reference system.

2. Determination of the absolute movements executed by the blank and, respectively, by the piece, during the generating process.
3. Determining the relative movements that occur between the tool and the blank.
4. Calculation of the trajectories described by the points belonging to the piece profile, while it performs the relative movement toward the tool.

5. Finding the enwrapping condition, condition that allows the choice of those points, belonging to the trajectories family, which, at the same time, also belong to the enveloping of this family.
6. Applying the enwrapping condition as a filter which allows the selection of the points belonging to the enveloping of the trajectories family generated during the relative movement of the piece toward the tool.

This enwrapping is precisely the searched profile of the tool.

The presented algorithm may prove to be difficult to apply especially because of the complexity of the calculations necessary to determine the relative movements between tool and piece.

In the following, a method is presented that allows avoiding the need to know the relative movement between the two involved elements, respectively the piece and the tool.

The new complementary method, called „the virtual pole method” starts from the premise that, in order to respect the Willis theorem [3], the normal at the profile to be generated, taken through the tangent point with the enveloping profile, must pass through the gearing pole.

Starting from this premise, we can deduce that, the profile to be generated, admits, in any point of it, a normal that, for a certain relative position of the two centrodes, will pass through the gearing pole. The respective moment is given by the position where the current point, through which the normal was traced, is in tangency with the enwrapping, so with the tool profile.

On the other hand, the gearing pole is defined as the tangency point of the two conjugated centrodes. So, if the normal to the profile $\Sigma$ (generated) taken through a current point passes through the gearing pole, logically, will also intersect the centrode associated with the profile $\Sigma$.

Therefore, an algorithm can be imagined to determine the intersection point between the normal at the profile $\Sigma$, taken through the current point and the centrode associated with $\Sigma$. This point was called the virtual pole.

Subsequently, this virtual pole is applied to the absolute movement that the tool performs, until its position coincides with the gearing pole.

In this position, the current point on $\Sigma$ is in contact with a point belonging to the active profile of the tool and at the same time belongs to the gearing curve. Being defined the gearing curve in the fixed system through its transfer into the tool space, its profile is defined.

In this way the coordinates of the point on the tool profile can be determined which coincides with the current point on the piece profile.

## 2. THE PROFILING ALGORITHM

In order to determine the tool profile intended to process, by enveloping, by the rolling method, a profile $\Sigma$ known in analytical or discrete form (by measurement), 3 reference systems are considered: $xy$and $x_0y_0$ are fixed reference systems; $XY$ - the mobile reference system, joined with the piece; $\xi\eta$ - the mobile reference system, joined with the tool.

The following notations will be used: $C_1$ - centrode of the piece; $C_2$ - centroid of the tool; $P_v$ - virtual pole; $T$ - current point; $P$ - gearing pole, the tangency point of the rolling centrodes; $\Sigma$ - generated profile; $S$ - profile to be generated (reciprocally enveloping with Σ); $CC$ - gearing curve.

### *2.1. Gear shaped cutter tool profiling*

In the case of the gear shaped cutter tool, the absolute movements of the piece and the tool are rotational movements, see Figure 1.

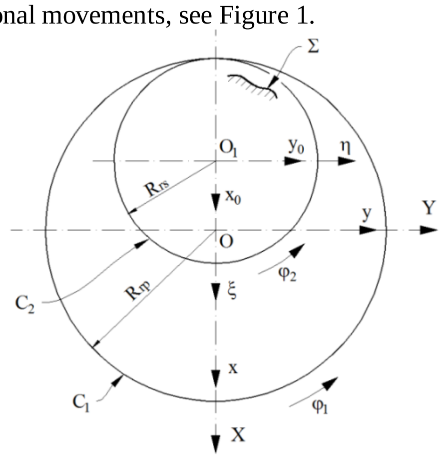


**Fig. 1.***Generating with gear shaped cutter. Rolling centrodes and reference systems*

The rolling condition between the two centrodes, both of circle type, is the condition that they roll without sliding:

$$R_{rp}\cdot\varphi_1 = R_{rs}\cdot\varphi_2 \Leftrightarrow \varphi_2 = \frac{R_{rp}}{R_{rs}}\cdot\varphi_1 = i\cdot\varphi_1. \quad (1)$$

In(1)it was noted with $i = R_{rp}/R_{rs}$ the transmission ratio.

The profile to be generated is considered known by the parametric equations:

$$\Sigma : \begin{vmatrix} X = X(u); \\ Y = Y(u), \end{vmatrix} \quad (2)$$

$u$,being the variable parameter.

In the case of profiles of line segment type, $u$ is considered as the distance measured from a defining point of the profile (one of the ends of the segment or its middle) to the current point.

In the case of profiles of circle arc type, $u$ is the angle that the radius makes through the current point with the horizontal or vertical direction, see Figure 2.

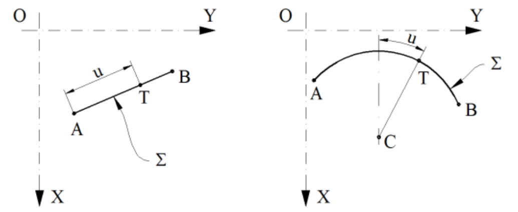


**Fig. 2.***Examples of the variable parameter, on Σ, for rectilinear or circular profiles*

As shown in the previous section, the virtual pole is defined as the intersection point intersection between the normal profile *Σ*, taken through the current point *T* and the centrode associated with the piece which, in this case, is a circle of radius $R_{rp}$.

The normal vector at profile *Σ*, having the module *λ*, is given by the equation:

$$\vec{N}_{\Sigma}=\lambda\cdot\left(\dot{Y}_u\cdot\vec{i}-\dot{X}_u\cdot\vec{j}\right). \quad (3)$$

The position vector of the current point *T* is given by:

$$\vec{r}=X(u)\cdot\vec{i}+Y(u)\cdot\vec{j}. \quad (4)$$

By summing the two vectors $\vec{r}$ and $\vec{N}_{\Sigma}$, the position vector of the virtual pole is obtained, see Figure 3.

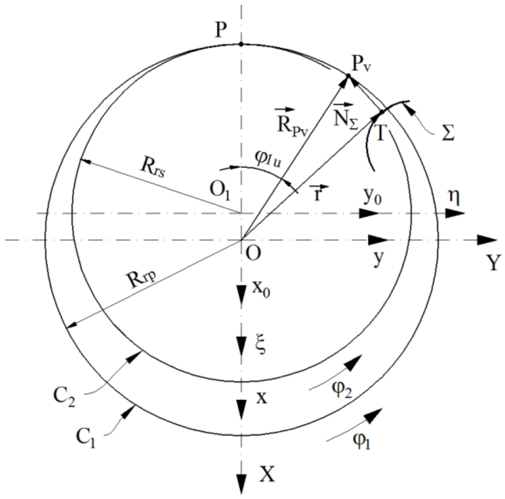


**Fig. 3.***Generating with inner rotary cutter. Rolling centrodes*

$$\begin{aligned}\vec{N}_{P_v}&=\vec{r}+\vec{N}_{\Sigma}=X(u)\cdot\vec{i}+Y(u)\cdot\vec{j}+\\&+\lambda\cdot\left(\dot{Y}_u\cdot\vec{i}-\dot{X}_u\cdot\vec{j}\right)=\left[X(u)+\lambda\cdot\dot{Y}_u\right]\cdot\vec{i}+\\&+\left[Y(u)-\dot{X}_u\cdot\lambda\right]\cdot\vec{j}.\end{aligned} \quad (5)$$

Being known the parametric equations of the centrode $C_1$:

$$C_1:\left|\begin{aligned}X&=-R_{rp}\cdot\cos\varphi_1;\\Y&=R_{rp}\cdot\sin\varphi_1,\end{aligned}\right. \quad (6)$$

it can be determined the value of the scalar *λ*, for which the definition of the $P_v$ pole is respected.

$$\left|\begin{aligned}X&=-R_{rp}\cdot\cos\varphi=X(u)+\lambda\cdot\dot{Y}_u;\\Y&=R_{rp}\cdot\sin\varphi=Y(u)-\lambda\cdot\dot{X}_u.\end{aligned}\right. \quad (7)$$

From the equations system (7)the parameter *λ* can be eliminated:

$$\lambda=\frac{-R_{rp}\cdot\cos\varphi_1-X(u)}{\dot{Y}_u}=\frac{-R_{rp}\cdot\sin\varphi_1+Y(u)}{\dot{X}_u}. \quad (8)$$

Equation (8)reduced to the form:

$$\begin{aligned}&\left[-R_{rp}\cdot\cos\varphi_1-X(u)\right]\cdot\dot{X}_u=\\&=\left[-R_{rp}\cdot\sin\varphi_1+Y(u)\right]\cdot\dot{Y}_u,\end{aligned} \quad (9)$$

represents precisely the enwrapping condition, that is, a connection between the independent parameters *u* and $\varphi_1$.

Therefore, for the current value of parameter *u*, the value $\varphi_{1u}$ given by (9)represents the angle at which the blank must be rotated, so that the virtual pole occupies the position *P*. By this rotation, of angle $\varphi_{1u}$, the current point *T*, will reach the position $T_F$ in the fixed reference system:

$$x=\omega_3^T\left(\varphi_{1u}\right)\cdot X, \quad (10)$$

or, developed:

$$\begin{aligned}\begin{pmatrix}x\\y\end{pmatrix}&=\begin{pmatrix}\cos\varphi_{1u}&-\sin\varphi_{1u}\\\sin\varphi_{1u}&\cos\varphi_{1u}\end{pmatrix}\cdot\begin{pmatrix}X(u)\\Y(u)\end{pmatrix}=\\&=\begin{pmatrix}X(u)\cdot\cos\varphi_{1u}-Y(u)\cdot\sin\varphi_{1u}\\X(u)\cdot\sin\varphi_{1u}+Y(u)\cdot\cos\varphi_{1u}\end{pmatrix}.\end{aligned} \quad (11)$$

The $T_F$-point, in the *xy* system, has the coordinates:

$$T_F:\left|\begin{aligned}x&=X(u)\cdot\cos\varphi_{1u}-Y(u)\cdot\sin\varphi_{1u};\\y&=X(u)\cdot\sin\varphi_{1u}+Y(u)\cdot\cos\varphi_{1u},\end{aligned}\right. \quad (12)$$

and will belong to the gearing curve.

At this moment, the *T* and $T_F$ points coincide and, moreover, are on the profile *S* of the tool. Their coordinates, in the reference system associated with the tool, can be determined by transforming coordinates corresponding to the absolute movement of the tool:

$$\xi=\omega_3^T\left(\pm\varphi_2\right)\cdot x; \quad (13)$$

„+" for internal generating; „-" for external generating.

Developing the equation (13)is obtained:

$$\begin{aligned}\begin{pmatrix}\xi\\\eta\end{pmatrix}&=\begin{pmatrix}\cos(\pm\varphi_2)&-\sin(\pm\varphi_2)\\\sin(\pm\varphi_2)&\cos(\pm\varphi_2)\end{pmatrix}\cdot\begin{pmatrix}x_o\\y_o\end{pmatrix}=\\&=\begin{pmatrix}x_o\cdot\cos(\pm\varphi_2)-y_o\cdot\sin(\pm\varphi_2)\\x_o\cdot\sin(\pm\varphi_2)+y_o\cdot\cos(\pm\varphi_2)\end{pmatrix}\end{aligned} \quad (14)$$

namely:

$$\left|\begin{aligned}\xi&=x_o\cdot\cos(\pm\varphi_2)-y_o\cdot\sin(\pm\varphi_2);\\\eta&=x_o\cdot\sin(\pm\varphi_2)+y_o\cdot\cos(\pm\varphi_2).\end{aligned}\right. \quad (15)$$

representing the transposition of the gearing curve in the tool system, so the profile of the gear shaped cutter.

Considering the transformation of coordinates between *x* and $x_o$:

$$x_o=x-A;\quad A=\begin{pmatrix}-A_{12}\\0\end{pmatrix};\quad -A_{12}=R_{rp}\pm R_{rs}, \quad (16)$$

$$\left|\begin{aligned}&\xi=\left(x+A_{12}\right)\cdot cos\left(\pm\varphi_2\right)-y_o\cdot sin\left(\pm\varphi_2\right);\\&\eta=\left(x+A_{12}\right)\cdot sin\left(\pm\varphi_2\right)+y_o\cdot cos\left(\pm\varphi_2\right).\end{aligned}\right. \tag{17}$$

So, the $T_S$ point on the tool profile and which coincides with the $T_F$-point on the gearing curve and with the $T$-point on the profile $\Sigma$ will have the coordinates:

$$T_S:\left|\begin{aligned}&\xi_{T_S}=\left(x_{T_F}+A_{12}\right)\cdot cos\left(\pm\varphi_2\right)-y_{T_F}\cdot sin\left(\pm\varphi_2\right);\\&\eta_{T_S}=\left(x_{T_F}+A_{12}\right)\cdot sin\left(\pm\varphi_2\right)+y_{T_F}\cdot cos\left(\pm\varphi_2\right).\end{aligned}\right. \tag{18}$$

### *2.2. Rotary cutter tool profiling*

In the case of the rotary cutter, the absolute movement of the tool is a rotational movement and the absolute movement of the piece is a translational movement.

In this case, the $C_1$ centrode associated with the piece is a line, while the $C_2$centrode, associated with the tool, is a radius circle $R_{rp}$.

The enwrapping condition is, in this case:

$$\delta=R_{rs}\cdot\varphi_1. \tag{19}$$

The fixed reference system will have the $x$-axis parallel to the $X$-axis and $y$ superimposed on the $Y$-axis, see Figure 4.

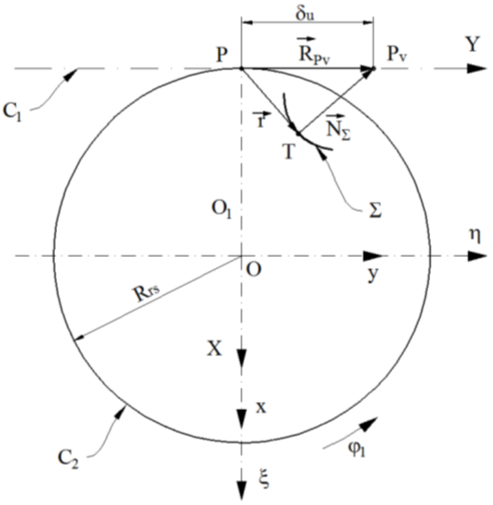


**Fig. 4.***Generating with rotary cutters. Reference systems and rolling centrodes*

As in the previous case, we consider the profile $\Sigma$ known by the parametric equations (2).

The parametric equations of the $C_1$ centrode are given by:

$$C_1:\left|\begin{aligned}&X=0;\\&Y=\delta.\end{aligned}\right. \tag{20}$$

From (5)and (20)the parameter $\lambda$ is eliminated:

$$\left|\begin{aligned}&X=X(u)+\lambda\cdot\dot{Y}_u=0;\\&Y=Y(u)-\lambda\cdot\dot{X}_u=\delta.\end{aligned}\right. \tag{21}$$

$$\lambda=\frac{-X(u)}{\dot{Y}_u}=\frac{-\delta+Y(u)}{\dot{X}_u}. \tag{22}$$

The specific form of the enwrapping condition will be:

$$-X(u)\cdot\dot{X}_u=\left[-\delta+Y(u)\right]\cdot\dot{Y}_u. \tag{23}$$

The $\delta_u$ value represents the translation value of the $XY$ system, associated to the part, which ensures the position of the $P_v$-point in the gearing pole. For this movement, the current point $T$ will have the coordinates given by its absolute movement:

$$x=X+A;\quad A=\begin{pmatrix}R_{rs}\\-\delta\end{pmatrix}, \tag{24}$$

or, developed:

$$T_F:\left|\begin{aligned}&x_{T_F}=X_T+R_{rs};\\&y_{T_F}=Y_T-\delta_u.\end{aligned}\right. \tag{25}$$

The $T_S$-point on the tool profile and which coincides with the $T$-point on the profile $\Sigma$ and with the $T_F$-point on the gearing curve will have the coordinates given by the absolute movement of the tool which is the rotation movement with the equations:

$$\begin{aligned}\begin{pmatrix}\xi\\\eta\end{pmatrix}&=\begin{pmatrix}cos\,\varphi_1 & sin\,\varphi_1\\-sin\,\varphi_1 & cos\,\varphi_1\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}=\\&=\begin{pmatrix}x\cdot cos\,\varphi_1+y\cdot sin\,\varphi_1\\-x\cdot sin\,\varphi\;+y\cdot cos\,\varphi_1\end{pmatrix}.\end{aligned} \tag{26}$$

Taking into account the enwrapping condition (19), which can be also written as $\varphi=\dfrac{\delta}{R_{rs}}$, is obtained:

$$T_S:\left|\begin{aligned}&\xi_{T_S}=x_{T_F}\cdot cos\left(\frac{\delta_u}{R_{rs}}\right)+y_{T_F}\cdot sin\left(\frac{\delta_u}{R_{rs}}\right);\\&\eta_{T_S}=-x_{T_F}\cdot sin\left(\frac{\delta_u}{R_{rs}}\right)+y_{T_F}\cdot cos\left(\frac{\delta_u}{R_{rs}}\right).\end{aligned}\right. \tag{27}$$

representing the transfer of the gearing curve in the $\xi\eta$ system meaning the analytical form of the rotary cutter.

## 3. CONCLUSIONS

This paper presented a new complementary method of profiling the tools of gear shaped cutter and rotary cutter type, namely “the virtual pole method”.

The method is based on a reinterpretation of the Willis theorem and allows eliminating the need to calculate the relative movements between tool and piece. This greatly simplifies the calculations necessary for profiling the types of tools mentioned, given that the determination of relative tool-piece movements is relatively complicated and can be a source of major errors.

In the future, it is intended to apply the “virtual pole method” for the elaboration of software products that allow the profiling of the tools that generate by enwrapping, by the rolling method, with applicability to profiles known both analytically and discreetly.

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